Total Pages-04 RNLKWC/B.Sc./CBCS/VIS/H/MTMH DSE601/22

2022

MATHEMATICS

[HONOURS]

(B.Sc. Sixth Semester End Examination-2022)

PAPER-MTMH DSE 601

[NUMBER THEORY]

Full Marks: 60 ***Time: 03Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Group A

1. Answer any ten questions: **2x10= 20**

i) Find the general solution in integer of the equation $8x-27y=1$.

ii) Show that the remainder when $6.7^{32}+7.9^{45}$ divided by 4 is 1

iii) Prove that the eighth power of any integer, is of the form $17k \text{ or } 17k \pm 1$

iv) If gcd (a,b)=1 then prove that gcd $(a+b, a^2-ab+b^2)=1$ or 3.

v) If p be a prime and k is a positive integer, then prove that $\phi(p^k) = p^k\left(1-\frac{1}{p}\right)$.

vi) If n is a positive integer such that $(n-1)! \equiv -1 \pmod{n}$,prove that n is prime.

vii) Find the number of zeros at the right end of the integer 141 !

viii) Find all primes p such that $\left(\frac{10}{p}\right)=1$

ix) Find the solution of the congruence $353x \equiv 254 \pmod{400}$

x) Let p be a prime number. Prove that $x^2 \equiv 1 \pmod{P}$ if and only if $x \equiv \pm 1 \pmod{p}$

xi) Show that if $d|m$,then $\phi(d)|\phi(m)$

xii) Show that 4(29)!+5! is divisible by 31.

xiii) Show that $2^{41} \equiv 3 \pmod{23}$

xiv) Determine the integer is the unit place of $19^{19^{19}}$

xv) Let n be a positive integer such that gcd (n,9)=1. Prove that 9 divides $n^{18}-1$.

Group B

2. Answer any four questions: 4x5 = 20

i) Prove that every prime number has a primitive root. Find $\pi(155)$.

ii) If $ax \equiv ay \pmod{m}$. then prove that $x \equiv y\left(\bmod \frac{m}{(a,m)}\right)$.

Show that $61!+1 \equiv 63!+1 \equiv 0 \pmod{71}$

iii) State and prove Chinese Remainder theorem.

iv) Prove that $53^{100}+103^{53}$ is divisible by 39.

v) State Mobius inversion formula. If P_n is the nth prime, then prove that $\frac{1}{p_1}+\frac{1}{p_2}+\ldots+\frac{1}{p_n}$ is not an integer.

vi) Find the least positive integer which leaves remainder 2,3 and 4 when divided by 3,5 and 11 respectively.

Find integer m,n such that $gcd\,(19,85)= 19m+85n$.

3. Answer any two questions: 2x10 = 20

i) a) Solve the system of congruences.

$x \equiv 14 \pmod{29}, x \equiv 5 \pmod{11}\quad x \equiv 15 \pmod{31}$.

b) If 2n+1 is prime, prove that $(n!)^2 \equiv (-1)^{n+1} \pmod{2n+1}$

ii) a) Let n be an integer greater than 1 such that $n = p_1^{r_1} p_2^{r_2} \ldots p_k^{r_k}$ where, $p_1, p_2, \ldots, p_k$ are distinct prime integers. Then prove that

$$\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_k}\right)$$

$\phi(6480)$

b) Define linear Diophantine equation. Let a,b,c,d be integers and m be a positive integer. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then prove that, $ax+by \equiv cx+dy \pmod{m}$

iii) a) If p and q are distinct primes and a is any integer, prove that $a^{pq} - a^{p} - a^{q} + a$ is divisible by pq. 3

b) Let $n > 2$ be an integer. Prove that $\phi(n)$ is even. 2

c) Use the theory of congruencies to find the remainder when the sum $1^5+2^5+3^5+\ldots+100^5$ is divisible by 5. 3

d) Prove that cube of any integer is of the form $9K\ or\ 9K \pm 1$. 2
