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2022

MATHEMATICS

[HONOURS]

(B.Sc. Sixth Semester End Examination-2022)

PAPER-MTMH DSE-602

[MATHEMATICS MODELLING]

Full Marks: 60 ***Time: 03Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any ten questions: **10x2= 20**

a) Prove that $P_n'(1) = \frac{1}{2}n(n+1)$

b) What is Bessel's differential equation? Specify the occurrence of this equation.

c) Prove that $J_{\frac{-3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{-\cos x}{x} - \sin x\right)$

d) Express $2-3x+4x^2$ in terms of Legendre polynomial.

e) Find the inverse Laplace transform of $\frac{s}{(s^4+4a^4)}$

f) Find the leplace transform of $\frac{e^{-t}\sin t}{t}$

g) Prove that $\frac{d}{dx}\left\{x^n J_n(x)\right\} = x^n J_{n-1}(x)$

h) Write the advantages and disadvantages of Mante Carlo method.

i) Write the Middle-square method for generating random number.

j) How the velocity of a car is related to traffic density?

k) Show that $L^{-1}\left(\frac{1}{s}\sin\frac{1}{s}\right) = t - \frac{t^3}{(3!)^3} + \frac{t^5}{(5!)^2} \ldots\ldots$

l) Prove that $P_n(-1) = (-1)^n$

m) In a harbor system, state the assumptions.

n) Why L.P.P requires sensitivity analysis?

o) Find $L\left[\int_0^t \frac{\sin t}{t}\, dt\right]$

2. Answer any four questions: **4x5 = 20**

a) Prove that $P_n(x) = \frac{1}{\pi}\int_0^{\pi} \frac{d\vartheta}{\left(x \pm \sqrt{x^2-1}\ \cos\theta\right)^{n+1}}$ where n being a positive integer.

b) Prove that $\int_{-1}^{1} P_m(x) P_n(x)\, dx = \begin{cases} 0 \ if \ m \neq n \\ \frac{2}{2n+1} \ if \ m = n \end{cases}$

c) Prove that $J_n\, J'_{-n} - J'_n\, J_{-n} = \frac{-2\sin \pi x}{x\pi}$

d) Using comolution theorem find $L^{-1}\left\{\frac{1}{s\left(s^2+4\right)}\right\}$

e) Evalute the integral $\int_0^1 e^{-x} dx$ using Monte-Carlo method.

f) Write down the Harbor system algorithm.

3. Answer any two questions of the following: **2x10 = 20**

a) i) solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$ with $x(0) = 1$ and $x\left(\frac{\pi}{2}\right) = -1$

ii)Using Laplace transform, solve $\left(d^4 - K^4\right)y = 0$, where $y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 0$

b) i) Establish and solve the governing equation for the probability of n-units in the queue system.

ii) Use Monte Carlo simulation to approximate the area under the curve $f(x) = \sqrt{x}$ over the interval $\frac{1}{2} \leq x \leq \frac{3}{2}$ 5+5

c) i) What do you understand by simulation? Explain briefly its limitation and advantages too?

ii) Optimize Z=6x+4y

subject to $-x + y \leq 12$

$x + y \leq 24$

$2x + 5y \leq 80,\ x \geq 0,\ y \geq 0$ 5+5