2021

Applied Mathematics with Oceanology and Computer Programming

[P.G.]

(CBCS)

(M.Sc. Third Semester End Examinations-2021)

MTM – 301

PARTIAL DIFFERNTIAL EQUATION AND GENERALISED FUNCTION

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Answer question No. 1 and FOUR from the rest.

1. **Answer any FOUR questions** **4x2=8**

   a) Define domain of dependence of one dimensional wave equation.

   b) Find the solution of $z^2 = pqxy$

   c) Discuss the nature of the second order partial differential equation $\left(x^2-1\right)u_{xx}+2yu_{xy}-u_{yy}=0$

d) Show that the Neumann problem for the Poisson's equation has more than one solution.

e) Let $f(t)$ be any continuous function. Then show that $\int_{-\alpha}^{\alpha} \delta(t-a) f(t) dt = f(a)$, where $\delta(x)$ is the Dirac-delta function.

f) Find the adjoint of the differential operation $L(u) = u_{xx} + u_{tt} - u_t$

2. a) Solve $\left(x^2D^2 - 2xyDD' + y^2D'^2 - xD + 3yD'\right)u = 8\frac{y}{x}$

$$D \equiv \frac{\partial}{\partial x} \quad D' \equiv \frac{\partial}{\partial y}$$

b) Find the characteristics of the equation $pq = xy$ and determine the integral surface which passes through the curve $z = x,\ y = 0$

4+4

3. Obtain the canonical form of the equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$ and hence solve it. 8

4. Obtain the solution of the interior Neumann problem for a circle given by the PDE

$$\nabla^2 u = 0,\ 0 \le r \le a, 0 \le \theta \le 2\pi .$$

$$Bc: \frac{\partial u}{\partial n} = \frac{\partial u(a,\theta)}{\partial r} = g(\theta),\ r = a$$

8

5. a) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ satisfying the conditions :

i) $u = 0$, when $x = 0$ and 1 for all t.

ii) $u = \begin{cases} 2x, & 0 \le x \le 1/2 \\ 2(1-x), & \frac{1}{2} \le x \le 1 \ when\, t = 0 \end{cases}$

b) $\delta(t)$ is the Dirac delta function, then show that

i) $\delta(-t) = \delta(t)$ and

ii) Prove that $\delta(at) = \frac{1}{a}\delta(t) \quad a > 0$

(2+2)+4

6. Solve $u_{tt} = c^2u_{xx}$, $0 \le x \le l$, $t > 0$

a) Subject to

$u(0,t) = 0,\ u(l,t) = 0$ for all t

$u(x,0) = 0,\ u_t(x,0) = b\, Sin^3(\pi x/l)$

b) $u(x,t)$ be the solution to the IVP

$u_{tt} = u_{xx}$, for $-\alpha < x < \alpha, t > 0$

With $u(x,0) = \sin x, u_t(x,0) = \cos x$,

Then find the value of $u(\pi/2,\ \pi/6)$

4+4

7. a) If the Neumann problem for a bounded region has a solution, then it is either unique or it differs from one another by a constant only.

(4)

b) Show that the Green's function $G\left(\overline{r},\overline{r}^{1}\right)$ has the symmetric property. 4+4

[Internal Marks – 10]