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ORIGINAL RESEARCH

Energy of interval-valued fuzzy graphs and its application in ecological systems

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Abstract

Many papers have been published for finding eigenvalues and eigenvectors of a fuzzy matrix (FM) (i.e. matrix with membership values) as well a fuzzy graph. But, all papers determine such parameters based on conventional arithmetic operators, though the valid operations on FMs are max–min. To the best of our knowledge no papers are published to find eigenvalues and eigenvectors based on max–min operators. In this paper, a novel technique is adapted to find the eigenvalues and eigenvectors of an interval-valued fuzzy graph (IVFG) using max–min operators. The energy of an IVFG is defined and computed using max–min operators. Finally, an application of eigenvalues of an IVFG is discussed for the ecological system. In ecology, the amount of food consumed by a predator from the preys is represented as an interval valued fuzzy membership values which is natural as the consumption of food for a predator from preys is uncertain. So this application is very much appropriate for eigenvalues and eigenvectors as well as energy of an IVFG.

Keywords Interval-valued fuzzy graph · Interval-valued fuzzy matrix · Fuzzy eigenvalues · Fuzzy eigenvectors · Energy of fuzzy graph · Food web

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1 Introduction

By the definition of fuzzy relations, it is familiar that real-life matters/objects be related to each other by a certain degree/grade. In real-life position, one object is either related or not related with another object. So, there is no opportunity to mention the strength or degree of relationship. But, we can assign the strength/degree of relationship, using the concept of fuzzy set theory between two objects. To represent a relationship there may be uncertainty/hesitation but, fuzzy set theory is enough if there is no hesitation or doubt to determine the strength/degree of relationship.

In extension of fuzzy sets there is a growing interest which can model not only uncertainty but also vague information. We know that the membership value completely depends on the thoughts of the decision maker, his habits, mentality, etc. So, membership value can't be measured as a point, but measured appropriately as an interval. Therefore, an interval-valued fuzzy set (IVFS) was successfully used to overcome the hesitation.

It is well known that graph representations are very important to model and analyze various real life problems and complex systems. Generally it is used in different domains for dealing with structural information such as operations research, computer science, engineering, networks, medical sciences, system analysis, ecosystems, image interpretation, pattern recognition and economics. One of the complex networks in the ecosystem is food web.

Eigenvalues and eigenvectors are generally crucial for determining the long term behavior of all kinds of models. A particular list of applications includes the followings:

- Population dynamics (biology/epidemiology)
- Stabilizing a system eg. anti lock brakes (control theory/engineering)
- Finding resonant frequencies (engineering, physics)
- Ranking internet pages by importance (computer science)
- Principal component analysis (statistics)
- Finding stable axes of rotation (physics)

Energy of a graph is the sum of absolute values of the eigenvalues of the adjacency matrix of that graph. This quantity is studied in the context of spectral graph theory. The energy of a given molecular graph, in chemistry is interesting as its relation to the total number of π-electron of the molecule represent by that graph. A graph has zero energy if all of its vertices are isolated, while for a complete graph K_n with n entries has $2(n-1)$ energy.

1.1 Review of related work

Interval-valued fuzzy sets, apparently with Zadeh in 1975 first proposed by Sambue [25], who called them ϕ-floues functions with respect to the assignment of membership degrees, serve to capture a feature of uncertainty. The actual idea is to replace fuzzy [0, 1]-valued membership degrees by subintervals of the interval [0, 1] realized to

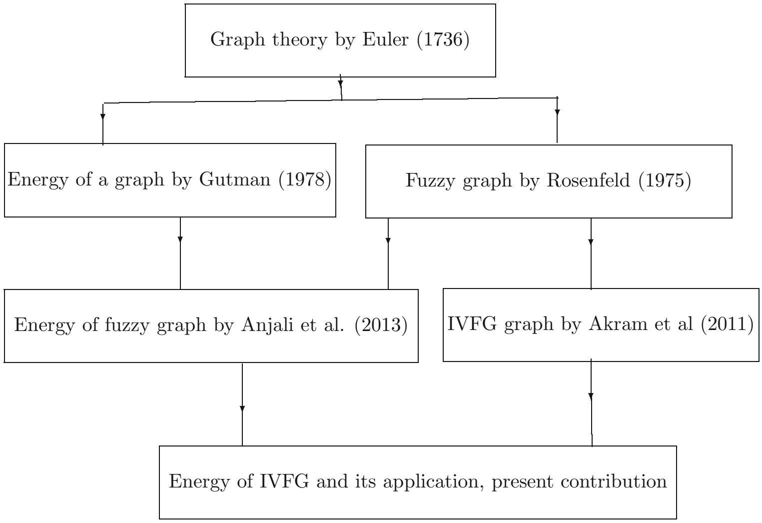


Fig. 1 Contribution chart

contain the true incompletely known degree of membership. As a generalization of fuzzy matrix (FM) the concept of IVFM was introduced and developed by Shyamal and Pal [26] in fuzzy algebra by extending the *max–min* operation. The interval-valued fuzzy vector space was first proposed in [15] and three types of rank viz, row rank, column rank and fuzzy rank of IVFM [18] are investigated and a relation is established between them. For more reading FM reader's are suggested to read the following [16,17,19,21]. Two new operators are defined by Shyamal and Pal [27] for fuzzy matrices.

Fuzzy eigenvalue problem was first proposed by Buckley in 1990 [6]. After that many authors tried [7,9,12] to find the eigenvalues and eigenvectors of fuzzy matrices. But, their techniques are not relevant for all kinds of matrices and those are very laborious methods. Some authors [7,9,12,31] are attempting to find out the eigenvalues and its eigenvectors as per regulation of crisp matrices by introducing the α-cut approach. In [13], an attempt has been made to find the eigenvalues and eigenvectors of fuzzy graph using max–min operations. But, no one used the *max–min* operations to find the eigenvalues and eigenvectors of interval-valued fuzzy graphs. Therefore, it was seen that the eigenvalues and associated eigenvectors were also negative [2] . But, for fuzzy sense negative values are not acceptable. We investigated the intuitionistic [16] and bipolar [19] fuzzy eigenvalues and associated eigenvectors of respective matrices by using *max–min* operation and obtained positive eigenvalues. For a real life application of graphs see [24]. The reader may consult with the recent book on fuzzy graph theory for the terminologies and applications of fuzzy graphs [22].

The interval-valued fuzzy graph is introduced by Akram et al. [1,3]. Formal concept analysis is described in [28] using interval-valued fuzzy formal concept lattice.

In Fig. 1, the contribution of fuzzy graphs and eigenvalues and eigenvectors of IVFG are depicted.

1.2 Motivation

From different extensions of fuzzy sets taking IVFS with graph theory create a new field called IVFG. We know that matrices are important tools to study/model different mathematical problems in linear algebra. Also imprecise/fuzzy data has huge applications in our real life aspect. One of the very important areas of fuzzy graphs is to find out the energy of the adjacency matrix and apply it in real fields. This is the motivation of our work.

1.3 Our contribution

In this article, a method is described to find the eigenvalues and eigenvectors of an IVFM and hence an IVFG using the max–min operators. The proposed method produces the eigenvalues which are the members of $D[0, 1]$ and this is natural. Then the energy of such a graph is computed and it is shown that the energy is also a member of $D[0, 1]$. Using these parameters of IVFG, the ecological system is investigated as a case study. An ecological system is modeled as an IVFG and based on the consumptions of food habits of the predators an IVFG is constructed. Then by finding eigenvalues and eigenvectors the energy is computed for the ecology.

1.4 Organization of the paper

The remaining part of the paper is organized as follows: in Sect. 2, the basic definitions of IVFM, IVFG and related terms are discussed. The methods for finding eigenvalues and eigenvectors are discussed in Sect. 3. Section 4 is devoted to compute the energy of an IVFG. In Sect. 5, an application of eigenvalues and eigenvectors in the ecological system is described. Also, a procedure to find the energy of an ecological system is provided in this section.

2 Preliminaries

In this section, some basic notions of the IVFM are recalled. To know about the IVFM first of all we have to know about the IVFS and interval-valued fuzzy algebra (IVFA).

Let $\mathbb{I} = \{a : 0 \leq a \leq 1\}$ be the set of all real numbers between 0 and 1. Also the set of all subsets of the interval [0, 1] be denoted by $D[0, 1]$ or simply D and is defined by

$$D[0, 1] = \{[a, b] : a \leq b;\ \ a, b \in \mathbb{I}\}$$

Definition 1 (*IVFS*) An IVFS A on the universe $U(\neq \phi)$ is given by

$$A = \{(x, A(x)) :\ x \in U\},\ \text{where } A(x) = [\underline{A}(x), \overline{A}(x)] \in D[0, 1].$$

Obviously, $A(x) = [\underline{A}(x), \overline{A}(x)]$ is the membership degree and $\underline{A}(x)$ is the lower limit, $\overline{A}(x)$ is the upper limit of the membership degree of $x \in U$ in A respectively.

The arithmetic operations like addition and multiplications of any two elements of D are as follows:

Definition 2 Let $u = [x_1, y_1]$ and $v = [x_2, y_2] \in D$. The addition (+) and multiplication $(\cdot)$ between u and v are defined below.

$$u + v = [x_1, y_1] + [x_2, y_2] = [max(x_1, x_2), max(y_1, y_2)] = [x_1 \vee x_2, y_1 \vee y_2]$$
$$\text{and } u \cdot v = [x_1, y_1] \cdot [x_2, y_2] = [min(x_1, x_2), min(y_1, y_2)] = [x_1 \wedge x_2, y_1 \wedge y_2].$$

Here it is seen that for the above operations only the values of lower and upper limits of membership degree are used. So an IVFS can be written as

$$A(x) = \left\{[\underline{x}, \overline{x}] : [\underline{x}, \overline{x}] \in D[0, 1]\right\},$$

where $\underline{x}, \overline{x}$ are the lower limits and upper limits of the membership degree of $x \in U$.

Now, two special elements are defined as follows:

Definition 3 (*Zero element*) The zero element of an IVFS is denoted and defined by $\emptyset = [0, 0]$.

Definition 4 (*Unit element*) The unit element of an IVFS is denoted and defined by $\ell = [1, 1]$.

Equality of two elements is defined as follow:

Definition 5 (*Equality*) Let $\mathbb{F}$ be an IVFS and $u, v \in \mathbb{F}$ where $u = [x_1, y_1]$ and $v = [x_2, y_2]$, then $u = v$ if and only if $x_1 = x_2$ and $y_1 = y_2$.

An IVFA is a mathematical system $(\mathbb{F}, +, \cdot)$ on the set $\mathbb{F}$ with two binary operations $+$ and $\cdot$ satisfying the properties written below.

(P1) Idempotent: $x + x = x, x \cdot x = x$
(P2) Commutativity: $x + y = y + x, x \cdot y = y \cdot x$
(P3) Associativity: $x + (y + z) = (x + y) + z, x \cdot (y \cdot z) = (x \cdot y) \cdot z$
(P4) Absorption: $x + (x \cdot y) = x, x \cdot (x + y) = x$
(P5) Distributivity: $x \cdot (y + z) = (x \cdot y) + (x \cdot z), x + (y \cdot z) = (x + y) \cdot (x + z)$
(P6) Universal bounds: $x + \emptyset = x, x + \ell = \ell, x \cdot \emptyset = \emptyset, x \cdot \ell = x$

where $x = [\underline{x}, \overline{x}],\ y = [\underline{y}, \overline{y}],\ z = [\underline{z}, \overline{z}]$ and $x, y, z \in \mathbb{F}$.

Definition 6 (*Interval-valued fuzzy vector*) An interval-valued fuzzy vector (IVFV) is an n-tuple of elements from an IVFA. That is, an IVFV is of the form $(a_1, a_2, a_3, \ldots, a_n)$, where each elements $a_i \in \mathbb{F},\ i = 1, 2, 3, \ldots, n$.

We denote $\mathbb{V}_n$ to be the set of all n-tuples $(a_1, a_2, a_3, \ldots, a_n)$ over $\mathbb{F}$. An element of $\mathbb{V}_n$ is known as IVFV of dimension n. Let $\mathbb{V}^n = \{\alpha^t \mid \alpha \in \mathbb{V}_n\}$ where the transpose of the vector α is denoted by α^t. Generally, an element of $\mathbb{V}_n$ is written as a row vector which is a $1 \times n$ matrix. The elements of $\mathbb{V}^n$ are column vectors.

Definition 7 (*IVFM*) An IVFM of order $m \times n$ is denoted by $M_{m\times n}$ which is the matrix over IVFA, i.e. $M_{m\times n} = (a_{ij})_{m\times n}$, where each $a_{ij} \in \mathbb{F}$.

The set of all rectangular matrices is denoted by $\mathbb{F}_{mn}$ of order $m \times n$ and the set of all square matrices is denoted by $\mathbb{F}_n$ of order n. The zero matrix $O_n \in \mathbb{F}_n$ is the matrix whose all elements are $\emptyset = [0, 0]$ and the identity matrix $\mathbb{I}_n \in \mathbb{F}_n$ is the matrix where all entries are $\emptyset = [0, 0]$ except the diagonal entries which are all $\ell = [1, 1]$.

3 Eigenvalues of interval-valued fuzzy matrices

In this section, a method to find the eigenvalues and eigenvectors of IVFMs by using *max–min* operation is discussed. Over crisp and fuzzy vector space [14], eigenvalue problems are useful in many areas. These problems are prepared when representing real cases into mathematical models. As for example, the principal axes in elasticity and dynamics, the natural frequencies and mode shapes in vibration problems, the analytical hierarchy process for decision making, and the Markov chain in stochastic modeling and queueing theory, etc. all are related to eigenvalue problems.

Definition 8 (*Eigenvalue and eigenvector*) Let a square matrix $A \in \mathbb{F}_n$ and a scalar $\xi = [\underline{\xi}, \overline{\xi}] \in \mathbb{F}$ be an eigenvalue of A and a vector Y (non-zero) known as column (row) eigenvector of A if $AY = \xi Y$ $(YA = \xi Y)$, where Y is defined as an eigenvector with respect to the eigenvalue ξ.

Theorem 1 *If $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$ be an IVFM of order $n\times n$, such that $a_{1i} = a_{2i} = \cdots = a_{i-1,i} = a_{i+1,i} = \cdots = a_{ni} = \emptyset$ (say) where $i = 1, 2, 3, \ldots, n$. Then a_{ii} is an eigenvalue associated to the column eigenvector $\kappa(\emptyset, \emptyset, \emptyset, \ldots, \ell, \ldots, \emptyset)^T \in \mathbb{V}^n$, where $\ell = [1, 1]$ be the ith entry and $\kappa \geq a_{ii}$.*

Proof Here $Y = \kappa(\emptyset, \emptyset, \emptyset, \ldots, \ell, \ldots, \emptyset)^T = (y_{i1})$ (say).

$$\text{Then } AY = \begin{pmatrix} \sum_{k=1}^{n} a_{1k}\cdot y_{k1} \\ \sum_{k=1}^{n} a_{2k}\cdot y_{k2} \\ \sum_{k=1}^{n} a_{3k}\cdot y_{k3} \\ \vdots \\ \sum_{k=1}^{n} a_{nk}\cdot y_{kn} \end{pmatrix} = \begin{pmatrix} \emptyset \\ \emptyset \\ \vdots \\ \kappa a_{ii} \\ \vdots \\ \emptyset \end{pmatrix} = a_{ii}\kappa \begin{pmatrix} \emptyset \\ \emptyset \\ \vdots \\ \ell \\ \vdots \\ \emptyset \end{pmatrix}$$

[Since for the ith entry $\sum_{k=1}^{n} a_{ik}\cdot x_{ki} = a_{i1}\cdot\emptyset + a_{i2}\cdot\emptyset + \cdots + a_{ii}\cdot\kappa + \cdots + a_{in}\cdot\emptyset = a_{ii}\kappa$.]

Therefore, $AY = a_{ii}Y$.

Hence, a_{ii} is the eigenvalue associating to column eigenvector $Y = \kappa(\emptyset, \emptyset, \ldots, \ell, \ldots, \emptyset)^T$. □

Example 1

$$\text{Let } A = \begin{pmatrix} [0.4, 0.5] & [0, 0] & [0.6, 0.7] \\ [0.5, 0.6] & [0.3, 0.4] & [0.5, 0.7] \\ [0.7, 0.8] & [0, 0] & [0.8, 0.9] \end{pmatrix} \text{ and } Y = \kappa([0, 0]\ \ [1, 1]\ \ [0, 0])^T$$
with $\kappa \geq [0.3, 0.4]$.

$$\text{Then } AY = \begin{pmatrix} [0.4, 0.5] & [0, 0] & [0.6, 0.7] \\ [0.5, 0.6] & [0.3, 0.4] & [0.5, 0.7] \\ [0.7, 0.8] & [0, 0] & [0.8, 0.9] \end{pmatrix} \kappa \begin{pmatrix} [0, 0] \\ [1, 1] \\ [0, 0] \end{pmatrix}$$
$$= \begin{pmatrix} [0, 0] \\ [0.3, 0.4] \\ [0, 0] \end{pmatrix} = [0.3, 0.4]\,\kappa \begin{pmatrix} [0, 0] \\ [1, 1] \\ [0, 0] \end{pmatrix} = [0.3, 0.4]\,Y.$$

Thus, [0.3, 0.4] is the eigenvalue of A associated to the column eigenvector Y.

Theorem 2 *Assume a square matrix $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$. If $a_{i1} = a_{i2} = \cdots = a_{i,i-1} = a_{i,i+1} = \cdots = a_{in} = \emptyset$ (say) where $i = 1, 2, 3, \ldots, n$. Then, a_{ii} is an eigenvalue associated to the row eigenvector $\kappa(\emptyset, \emptyset, \emptyset, \ldots, \ell, \ldots, \emptyset) \in \mathbb{V}_n$, where ℓ be the ith entry and $a_{ii} \leq \kappa$.*

Proof The proof is similar to Theorem 1. □

Example 2

$$\text{Let } A = \begin{pmatrix} [0.3, 0.6] & [0.6, 0.8] & [0.2, 0.3] \\ [0, 0] & [0.4, 0.6] & [0, 0] \\ [0.4, 0.5] & [0.5, 0.6] & [0.8, 0.9] \end{pmatrix} \text{ and } Y = \kappa([0, 0]\ \ [1, 1]\ \ [0, 0])$$
with $[0.4, 0.6] \leq \kappa$.

$$\text{Then } YA = \kappa([0, 0]\ \ [1, 1]\ \ [0, 0]) \begin{pmatrix} [0.3, 0.6] & [0.6, 0.8] & [0.2, 0.3] \\ [0, 0] & [0.4, 0.6] & [0, 0] \\ [0.4, 0.5] & [0.5, 0.6] & [0.8, 0.9] \end{pmatrix}$$
$$= ([0, 0]\ \ [0.4, 0.6]\ \ [0, 0]) = [0.4, 0.6]\,\kappa([0, 0]\ \ [1, 1]\ \ [0, 0]).$$

Therefore, $YA = [0.4, 0.6]\,Y$.

Hence, [0.4, 0.6] is the eigenvalue of A associated to the row eigenvector Y.

Theorem 3 *Consider a square matrix $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$ such that $a_{ij} \leq a_{1i} = a_{2i} = a_{3i} = \cdots = a_{ni} = \xi \leq \kappa$ for all $i, j = 1, 2, 3, \ldots, n$. Then, ξ is an eigenvalue associated to the column eigenvector $\kappa(\ell, \ell, \ell, \ldots, \ell)^T \in \mathbb{V}^n$.*

Proof Since $a_{ij} \leq a_{1i} = a_{2i} = a_{3i} = \cdots = a_{ni} = \xi \leq \kappa$ for all $i, j = 1, 2, 3, \ldots, n$.

Therefore, $\sum_{j=1}^{n} a_{ij} = \xi$. Also $Y = \kappa(\ell, \ell, \ell, \ldots, \ell)^T$.

$$\text{Then } AY = \kappa \begin{pmatrix} \sum_{j=1}^{n} a_{1j}\ell \\ \sum_{j=1}^{n} a_{2j}\ell \\ \vdots \\ \sum_{j=1}^{n} a_{nj}\ell \end{pmatrix} = \kappa \begin{pmatrix} \sum_{j=1}^{n} a_{1j} \\ \sum_{j=1}^{n} a_{2j} \\ \vdots \\ \sum_{j=1}^{n} a_{nj} \end{pmatrix} = \kappa \begin{pmatrix} \xi \\ \xi \\ \vdots \\ \xi \end{pmatrix} = \xi\kappa \begin{pmatrix} \ell \\ \ell \\ \vdots \\ \ell \end{pmatrix} = \xi Y.$$

This shows, ξ is an eigenvalue of A associated to the column eigenvector Y. □

Example 3

$$\text{Let } A = \begin{pmatrix} [0.4, 0.5] & [0.8, 0.9] & [0.6, 0.7] \\ [0.7, 0.8] & [0.8, 0.9] & [0.5, 0.7] \\ [0.6, 0.7] & [0.8, 0.9] & [0.7, 0.8] \end{pmatrix} \text{ and } Y = \kappa([1, 1]\ [1, 1]\ [1, 1])^T$$

with $\kappa \geq [0.8, 0.9]$.

Then

$$AY = \begin{pmatrix} [0.4, 0.5] & [0.8, 0.9] & [0.6, 0.7] \\ [0.7, 0.8] & [0.8, 0.9] & [0.5, 0.7] \\ [0.6, 0.7] & [0.8, 0.9] & [0.7, 0.8] \end{pmatrix} \kappa \begin{pmatrix} [1, 1] \\ [1, 1] \\ [1, 1] \end{pmatrix} = \begin{pmatrix} [0.8, 0.9] \\ [0.8, 0.9] \\ [0.8, 0.9] \end{pmatrix}$$
$$= [0.8, 0.9]\,\kappa \begin{pmatrix} [1, 1] \\ [1, 1] \\ [1, 1] \end{pmatrix}.$$

Hence, $AY = [0.8, 0.9]\,Y$.
Thus, [0.8, 0.9] is the column eigenvalue of A associated to the eigenvector Y.

Theorem 4 *Consider a square matrix $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$ such that $a_{ij} \leq a_{i1} = a_{i2} = a_{i3} = \cdots = a_{in} = \xi \leq \kappa$ for all $i, j = 1, 2, 3, \ldots, n$. Then, ξ is an eigenvalue of A corresponding to the row eigenvector $\kappa(\ell, \ell, \ell, \ldots, \ell) \in \mathbb{V}_n$.*

Proof The proof is similar to the Theorem 3. □

Example 4

$$\text{Let } A = \begin{pmatrix} [0.5, 0.6] & [0.6, 0.8] & [0.7, 0.8] \\ [0.8, 0.9] & [0.5, 0.7] & [0.6, 0.7] \\ [0.9, 1] & [0.9, 1] & [0.9, 1] \end{pmatrix} \text{ and } Y = \kappa([1, 1]\ [1, 1]\ [1, 1])$$

with $\kappa \geq [0.9, 1]$.

$$\text{Then } YA = \kappa([1, 1]\ [1, 1]\ [1, 1]) \begin{pmatrix} [0.5, 0.6] & [0.6, 0.8] & [0.7, 0.8] \\ [0.8, 0.9] & [0.5, 0.7] & [0.6, 0.7] \\ [0.9, 1] & [0.9, 1] & [0.9, 1] \end{pmatrix}$$
$$= ([0.9, 1]\ [0.9, 1]\ [0.9, 1]) = [0.9, 1]\,\kappa([1, 1]\ [1, 1]\ [1, 1]).$$

Therefore, $YA = [0.9, 1]\,Y$.
Hence, [0.9, 1] is the eigenvalue of A associated to the row eigenvector Y.

Definition 9 (*Diagonally dominant*) Let $A = (a_{ij}) \in \mathbb{F}_n$ be a square IVFM of order n. A is called row diagonally dominant if $a_{ii} \geq \sum_{j\neq i, j=1}^{n} a_{ij}$, A is called column diagonally dominant if $a_{ii} \geq \sum_{i\neq j, i=1}^{n} a_{ij}$ and A is called diagonally dominant if and only if it is both row as well as column diagonally dominant.

Theorem 5 *Consider a square matrix $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$ such that $a_{11} = a_{22} = a_{33} = \cdots = a_{nn} = \xi \leq \kappa$ (say) and if A is diagonally dominant, then ξ is an eigenvalue corresponding to the row (column) eigenvector $\kappa(\ell, \ell, \ell, \ldots, \ell) \in \mathbb{V}_n$ $\left(\kappa(\ell, \ell, \ell, \ldots, \ell)^T \in \mathbb{V}^n\right)$.*

Proof Since the IVFM $A = (a_{ij})$ is diagonally dominant, therefore $\sum_{j=1}^{n} a_{ij} = a_{ii} = \xi \leq \kappa$. Also $Y = \kappa(\ell, \ell, \ell, \ldots, \ell)^T$. Then

$$AX = \kappa \begin{pmatrix} \sum_{j=1}^{n} a_{1j}\ell \\ \sum_{j=1}^{n} a_{2j}\ell \\ \vdots \\ \sum_{j=1}^{n} a_{nj}\ell \end{pmatrix} = \kappa \begin{pmatrix} \sum_{j=1}^{n} a_{1j} \\ \sum_{j=1}^{n} a_{2j} \\ \vdots \\ \sum_{j=1}^{n} a_{nj} \end{pmatrix} = \kappa \begin{pmatrix} \xi \\ \xi \\ \vdots \\ \xi \end{pmatrix} = \xi\kappa \begin{pmatrix} \ell \\ \ell \\ \vdots \\ \ell \end{pmatrix} = \xi Y.$$

Thus, ξ is an eigenvalue of the IVFM A corresponding to the column eigenvector Y.
We can prove the theorem similarly for row eigenvector. □

Example 5

$$\text{Let } A = \begin{pmatrix} [0.8, 0.9] & [0.5, 0.7] & [0.6, 0.8] \\ [0.7, 0.9] & [0.8, 0.9] & [0.5, 0.6] \\ [0.5, 0.5] & [0.7, 0.8] & [0.8, 0.9] \end{pmatrix} \text{ and } Y = \kappa([1, 1]\ [1, 1]\ [1, 1])$$
with $\kappa \geq [0.8, 0.9]$.

$$\text{Therefore, } XA = \kappa([1, 1]\ [1, 1]\ [1, 1]) \begin{pmatrix} [0.8, 0.9] & [0.5, 0.7] & [0.6, 0.8] \\ [0.7, 0.9] & [0.8, 0.9] & [0.5, 0.6] \\ [0.5, 0.5] & [0.7, 0.8] & [0.8, 0.9] \end{pmatrix}$$
$$= ([0.8, 0.9]\ [0.8, 0.9]\ [0.8, 0.9]) = [0.8, 0.9]\,\kappa([1, 1]\ [1, 1]\ [1, 1]).$$

That is, $YA = [0.8, 0.9]\,Y$.
Hence, [0.8, 0.9] is the eigenvalue of A corresponding to the row eigenvector Y.

Theorem 6 *If a square matrix $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$, then $\xi = (\underline{\xi}, \overline{\xi}) \leq \kappa \in \mathbb{F}$ be an eigenvalue associated to the column eigenvector $\kappa(\ell, \ell, \ell, \ldots, \ell)^T \in \mathbb{V}^n$ if $\max\{\underline{a}_{k1}, \underline{a}_{k2}, \underline{a}_{k3}, \ldots, \underline{a}_{kn}\} = \underline{\xi}$ and $\max\{\overline{a}_{k1}, \overline{a}_{k2}, \overline{a}_{k3}, \ldots, \overline{a}_{kn}\} = \overline{\xi}$ for every $k \in \{1, 2, 3, \ldots, n\}$.*

Proof Since $\max\{\underline{a}_{k1}, \underline{a}_{k2}, \underline{a}_{k3}, \ldots, \underline{a}_{kn}\} = \underline{\xi}$ and $\max\{\overline{a}_{k1}, \overline{a}_{k2}, \overline{a}_{k3}, \ldots, \overline{a}_{kn}\} = \overline{\xi}$ for every $k \in \{1, 2, 3, \ldots, n\}$. Therefore, $\sum_{j=1}^{n} a_{kj} = (\sum_{j=1}^{n} \underline{a}_{kj}, \sum_{j=1}^{n} \overline{a}_{kj}) = (\underline{\xi}, \overline{\xi}) = \xi$ for every $k \in \{1, 2, 3, \ldots, n\}$. Also $Y = (\ell, \ell, \ell, \ldots, \ell)^T \in \mathbb{V}^n$. Then

$$AY = \kappa \begin{pmatrix} \sum_{j=1}^{n} a_{1j}\ell \\ \sum_{j=1}^{n} a_{2j}\ell \\ \vdots \\ \sum_{j=1}^{n} a_{nj}\ell \end{pmatrix} = \kappa \begin{pmatrix} \sum_{j=1}^{n} a_{1j} \\ \sum_{j=1}^{n} a_{2j} \\ \vdots \\ \sum_{j=1}^{n} a_{nj} \end{pmatrix} = \kappa \begin{pmatrix} \xi \\ \xi \\ \vdots \\ \xi \end{pmatrix} = \xi\kappa \begin{pmatrix} \ell \\ \ell \\ \vdots \\ \ell \end{pmatrix} = \xi Y.$$

Thus, ξ is an eigenvalue of the IVFM A, and the corresponding column eigenvector is Y. □

Example 6

$$\text{Let } A = \begin{pmatrix} [0.5, 0.6] & [0.6, 0.8] & [0.7, 0.7] \\ [0.7, 0.8] & [0.6, 0.7] & [0.5, 0.6] \\ [0.7, 0.7] & [0.3, 0.4] & [0.5, 0.8] \end{pmatrix} \text{ and } Y = \kappa([1, 1]\ [1, 1]\ [1, 1])^T$$

with $[0.7, 0.8] \leq \kappa$.

$$\text{Then } AY = \begin{pmatrix} [0.5, 0.6] & [0.6, 0.8] & [0.7, 0.7] \\ [0.7, 0.8] & [0.6, 0.7] & [0.5, 0.6] \\ [0.7, 0.7] & [0.3, 0.4] & [0.5, 0.8] \end{pmatrix} \kappa \begin{pmatrix} [1, 1] \\ [1, 1] \\ [1, 1] \end{pmatrix} = \begin{pmatrix} [0.7, 0.8] \\ [0.7, 0.8] \\ [0.7, 0.8] \end{pmatrix}$$
$$= [0.7, 0.8]\kappa \begin{pmatrix} [1, 1] \\ [1, 1] \\ [1, 1] \end{pmatrix}.$$

Hence, $AY = [0.7, 0.8]\,Y$.
Thus, $[0.7, 0.8]$ is the column eigenvalue of A associated to the eigenvector Y.

Theorem 7 *If a square matrix* $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$, *then* $\xi = (\underline{\xi}, \overline{\xi}) \leq \kappa \in \mathbb{F}$ *be an eigenvalue associated to the row eigenvector* $\kappa(\ell, \ell, \ell, \ldots, \ell) \in V_n$ *if* $\max\{\underline{a}_{1k}, \underline{a}_{2k}, \underline{a}_{3k}, \ldots, \underline{a}_{mk}\} = \underline{\xi}$ *and* $\max\{\overline{a}_{1k}, \overline{a}_{2k}, \overline{a}_{3k}, \ldots, \overline{a}_{nk}\} = \overline{\xi}$ *for every* $k \in \{1, 2, 3, \ldots, n\}$.

Proof The proof is similar to the Theorem 6. □

Example 7

$$\text{Let } A = \begin{pmatrix} [0.8, 0.8] & [0.4, 0.5] & [0.6, 0.9] \\ [0.7, 0.9] & [0.8, 0.9] & [0.5, 0.6] \\ [0.5, 0.6] & [0.7, 0.8] & [0.8, 0.8] \end{pmatrix} \text{ and } Y = \kappa([1, 1]\ [1, 1]\ [1, 1])$$
with $[0.8, 0.9] \leq \kappa$.

$$\text{Therefore, } YA = \kappa([1, 1]\ [1, 1]\ [1, 1]) \begin{pmatrix} [0.8, 0.8] & [0.4, 0.5] & [0.6, 0.9] \\ [0.7, 0.9] & [0.8, 0.9] & [0.5, 0.6] \\ [0.5, 0.6] & [0.7, 0.8] & [0.8, 0.8] \end{pmatrix}$$
$$= ([0.8, 0.9]\ [0.8, 0.9]\ [0.8, 0.9]) = [0.8, 0.9]\,\kappa([1, 1]\ [1, 1]\ [1, 1]).$$

That is, $YA = [0.8, 0.9]\,Y$.
Hence, $[0.8, 0.9]$ is the eigenvalue of A associated to the row eigenvector Y.

Corollary 1 *Let a square matrix* $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$. *If* $\sum_{j=1}^{n} a_{1j} = \sum_{j=1}^{n} a_{2j} = \cdots = \sum_{j=1}^{n} a_{nj} = \xi \leq \kappa$ *(say). Then,* ξ *is an eigenvalue of A associated to the column eigenvector* $\kappa(\ell, \ell, \ell, \ldots, \ell)^T \in \mathbb{V}^n$.

Corollary 2 *Let a square matrix* $A = (a_{ij}) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{F}_n$. *If* $\sum_{i=1}^{n} a_{i1} = \sum_{i=1}^{n} a_{i2} = \cdots = \sum_{i=1}^{n} a_{in} = \xi \leq \kappa$ *(say). Then,* ξ *is an eigenvalue of A associated to the row eigenvector* $\kappa(\ell, \ell, \ell, \ldots, \ell) \in \mathbb{V}_n$.

Let $\sigma(A)$ denote the set of all eigenvalues of A, i.e. spectrum of the FM A.

Theorem 8 *Let* $A \in \mathbb{F}_n$ *be a square IVFM, then A has a zero column iff* $\emptyset \in \sigma(A)$.

Proof Let the ith column of A be zero and $Y = \kappa(\emptyset, \emptyset, \ldots, \ell, \ldots, \emptyset)^T$, where ℓ is the ith entry. Then Y is a non-zero vector satisfying the equation $AY = \emptyset Y = \emptyset$. Hence, Y is a column eigenvector corresponding to the eigenvalue $\emptyset$.

Let $Y = \kappa(y_1, y_2, y_3, \ldots, y_n)^T$ be a column eigenvector associating to the eigenvalue $\emptyset$, then $AY = \emptyset$. We assume that $y_i \neq \emptyset$ for $i \in \{1, 2, 3, \ldots, n\}$. Then $AY = \emptyset$ implies that $\sum_{k=1}^{n} a_{jk} \cdot y_k = \emptyset$ for each $j \in \{1, 2, 3, \ldots, n\}$. This implies $a_{jk} \cdot y_k = \emptyset$ for each j and k. Since $y_i \neq \emptyset$, $a_{ij} = \emptyset$ for each j, thus the ith column of A is zero. □

Definition 10 Let $\sigma(A)$ be the set of all eigenvalues of $A \in \mathbb{F}_n$. Then $\delta(A) = \sup\{\xi \mid \xi \in \sigma(A)\}$ is called the spectral radius of A.

Theorem 9 *Let* $A \in F_n$. *Then* $\delta(A)$ *is either* $\emptyset$ *or* ℓ.

Proof If $\sigma(A) = \{\emptyset\}$, then $\delta(A) = \emptyset$, otherwise, if there exist $\xi(\neq \emptyset) \in \sigma(A)$, then there is a non-zero eigenvector $Y \in \mathbb{V}^n$ such that $AY = \xi Y$. Also, we know that for any α with $\xi \leq \alpha \leq \ell$, $\alpha \cdot \xi = \xi$ and $\xi \cdot \xi = \xi$.

Therefore, $\xi Y = (\alpha \cdot \xi)Y = \alpha(\xi Y)$
Now, $A(\xi Y) = \xi(AY) = \xi(\xi Y) = (\xi \cdot \xi)Y = \xi Y = \alpha(\xi Y)$.
Hence, $\alpha \in \sigma(A)$.

Since α is arbitrary, $\ell \in \sigma(A)$. Therefore, $\delta(A) = \ell$. □

Theorem 10 *For any* $A, B \in \mathbb{F}_n$ *if* $A \leq B$, *then* $\delta(A) \leq \delta(B)$.

Proof From Theorem 2, $\delta(A)$ is either $\emptyset$ or ℓ.
If $\delta(A) = \emptyset$, then $\delta(A) \leq \delta(B)$ holds trivially.
If $\delta(A) = \ell$, we have to prove that $\delta(B) = \ell$.

Since $\delta(A) = \ell$, then by definition $\ell \in \sigma(A)$ and $AY = \ell Y = Y$ for some non-zero column vector Y. We consider $e = (\ell, \ell, \ell, \ldots, \ell)^T$, then $Y \leq e$.
Also $A^n Y = A^{n-1} AY = A^{n-1} Y = A^{n-2} Y = \cdots = A^2 Y = AY = Y$
i.e., $Y = A^n Y \leq A^n e \leq B^n e$. [Since $Y \leq e$ and $A \leq B$.]

Since Y is non-zero hence $B^n e$ is non-zero.
Now, if $Y = B^n e$, then $BY = B^{n+1} e = B^n e = Y = \ell Y$. Hence, $\ell \in \sigma(B)$.
Thus, $\delta(B) = \ell$. □

4 Energy of interval-valued fuzzy graph

In this section, the IVFG, interval-valued fuzzy digraph (IVFDG), adjacency matrix of IVFDG and energy of an IVFDG under interval-valued fuzzy circumstances are defined.

Definition 11 (*IVFG*) Let $G^* = (V, E)$ be a crisp graph with $E \subseteq V \times V$. An IVFG of a graph $G^* = (V, E)$ is a pair $G = (A, B)$, where $A = [\mu_{\underline{A}}, \mu_{\overline{A}}] \in V$ is an IVFS with condition $0 \leq \mu_{\underline{A}}(x) \leq \mu_{\overline{A}}(x) \leq 1$ for all $x \in V$ and $B = [\mu_{\underline{B}}, \mu_{\overline{B}}] \in E$ is an interval-valued fuzzy relation with conditions

$$\begin{aligned} \mu_{\underline{B}}(xy) &\leq \min\{\mu_{\underline{A}}(x), \mu_{\underline{A}}(y)\} \\ \text{and } \mu_{\overline{B}}(xy) &\leq \min\{\mu_{\overline{A}}(x), \mu_{\overline{A}}(y)\} \text{ for all } xy \in E. \end{aligned}$$

Here A be known as interval-valued fuzzy vertex set of V, and B be known as interval-valued fuzzy edge set of E.

Definition 12 (*IVFDG*) Let $\overrightarrow{G}^* = (V, \overrightarrow{E})$ be a crisp directed graph (diagraph) with directed edge $\overrightarrow{E} \subseteq V \times V$. An IVFDG of a graph $\overrightarrow{G}^* = (V, \overrightarrow{E})$ is a pair $\overrightarrow{G} = (A, \overrightarrow{B})$, where $A = [\mu_{\underline{A}}, \mu_{\overline{A}}] \in V$ is an IVFS with condition $0 \leq \mu_{\underline{A}}(x) \leq \mu_{\overline{A}}(x) \leq 1$ for all $x \in V$ and $\overrightarrow{B} = [\mu_{\underline{B}}, \mu_{\overline{B}}] \in \overrightarrow{E}$ is an interval-valued fuzzy relation with conditions

$$\begin{aligned} \mu_{\underline{B}}(\overrightarrow{xy}) &\leq \min\{\mu_{\underline{A}}(x), \mu_{\underline{A}}(y)\} \\ \text{and } \mu_{\overline{B}}(\overrightarrow{xy}) &\leq \min\{\mu_{\overline{A}}(x), \mu_{\overline{A}}(y)\} \text{ for all } \overrightarrow{xy} \in \overrightarrow{E}. \end{aligned}$$

Here A and B are known as interval-valued fuzzy vertex sets of V and interval-valued fuzzy edge sets of E respectively. A weighted IVFDG is a digraph associated with its edge weight as an interval number.

If a graph is large, then it is very difficult to handle without using a computer. To represent it in a computer, several methods are available, among them the adjacent matrix is one of the most useful techniques which is discussed below.

Definition 13 (*Adjacency matrix of weighted IVFDG*) Let $\overrightarrow{G} = (V, \overrightarrow{E})$ be a weighted IVFDG with weight $w_{ij} \in D[0, 1]$ of the directed edges from v_i to v_j i.e, $(\overrightarrow{v_i v_j})$ then the adjacency matrix of this directed graph is denoted by $M = (m_{ij})_n \in \mathbb{F}_n$, where n is the number of vertices of the graph and is defined by

$$m_{ij} = \begin{cases} w_{ij}; & \text{if } (\overrightarrow{v_i v_j}) \in \overrightarrow{E} \\ 0; & \text{otherwise} \end{cases}$$

Therefore, every IVFDG has an adjacency matrix and so it has some eigenvalues.

It is observed that for any IVFG, there is an adjacency IVFM. An outline is given to find the eigenvalues of an IVFM. So, for any IVFM, one can determine some eigenvalues.

Definition 14 (*Energy of IVFG*) Let $G = (V, E)$ be a weighted IVFG and $M = (m_{ij})$ be its adjacency matrix of order $n \times n$. Also, let $\xi_1, \xi_2, \xi_3, \ldots, \xi_n$ be the eigenvalues of the corresponding adjacency matrix of this weighted IVFG. Then, the energy of this IVFG is denoted by $\mathbb{E}(G)$ and is defined by

$$\mathbb{E}(G) = \sum_{i=1}^{n} |\xi_i|.$$

For an IVFG, all the eigenvalues are the members of D, so this definition is updated as

$$\mathbb{E}(G) = \sum_{i=1}^{n} [\underline{\xi}_i, \overline{\xi}_i].$$

5 Application to ecosystem

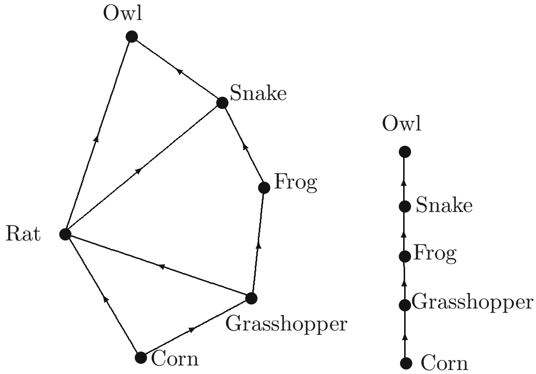
In an ecosystem, different types of species are there, each species depends for food on one or more other species, except producers (plants and some organic chemicals). The species are divided into two groups, preys and predators. A predator be prey for other predators. We can explain the prey-predator relationship easily with the help of a directed graph called a food web.

The primary producers are called basal species. The species that eat plants are known as herbivores or grazers. The animals that eat herbivores or each other are called carnivores or predators. Species that eat both plants and other animals are called omnivores. The basal species always stayed at the bottom of the food web.

For example, let us consider a small ecosystem, owls eat snakes and rats, snakes eat frogs and rats, frogs eat grasshoppers, rats eat grasshoppers and corn, grasshoppers eat corn. These relationships can be represented by a diagraph shown in Fig. 2 called food web.

The six organisms viz. corn, grasshopper, rat, frog, snake and owl are taken as vertices. There is an edge from species P to species Q if species Q preys on species P. In Fig. 2 corn is in the bottom of the food web and owl at the top, top predator. In a particular food web, if there is one and only one species is above the other then it is called the food chain. For example, the food web of Fig. 2b is a food chain.

Fig. 2 Some small food webs



(a) Food web **(b)** Food chain

In 1960, Cohen [10] constructed a food web to analyze ecosystems. Cattin et al. [8] proposed a new model assuming that any species diet is the consequence of phylogenetic constraints and adaptation. Cohen [10], observed that the food web constructed from homogeneous ecosystems generally have competition graphs that are interval graphs (a very simple graph structure and it has lots of application in many fields). But, it is not true for all food webs. Cohen et al. [11] showed that the probability that a competition graph is an interval graph tends to 0 as the number of species increases. After that many people are working on this area, for details see [4,5,20,23,29,30,32–34].

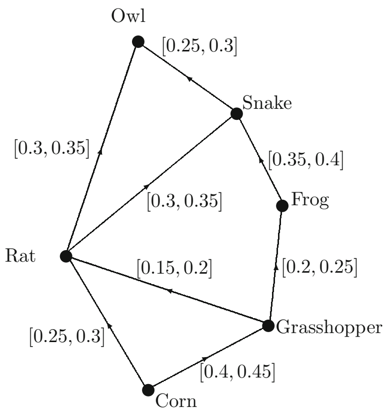
5.1 Weighted food web and weighted fuzzy graph

It is obvious that all the relations are not the same. For example, snakes eat rats and frogs, but snakes do not eat the same amount of rats and frogs. It may differ among species to species belonging to the same genus, e.g. snakes and frogs have different varieties. In general, species may eat much more of one species of prey then another depending on availability of food. Using the quadrat method for finding food habits of snakes with respect to the total biomass of a particular food web we observe that snakes eat rats (say 30–35%) and frogs (say 35–40%). These numbers can be assigned to the corresponding edges as weights. Thus, weights on the edges represent the food preferences with respect to the total biomass of the food habits. The weight of the directed edge $(\overrightarrow{ij})$ is denoted by w_{ij}, which represents the proportion of the food contribution of the vertex i to the vertex j.

5.2 Formation of adjacency matrix

To make the adjacency (relation) matrix of the above weighted food web (graph) first of all we have to draw the order of every vertex, i.e. species of the food web. Here, we correspond to the natural number for every element of the food web, which is taken

Fig. 3 Weighted food web



according to the following priority charts.

Highest consumer (if any)	
↑	
Tertiary consumer	Carnivorus
↑	↑
Secondary consumer	Omnivorus
↑	↑
Primary consumer	Herbivorus
↑	↑
Producer	Producer
Priority chart 1	**Priority chart 2**

Here, the first priority chart taken according as food chain and second priority chart taken according as food habits

If there is a tie up of two species for numbering then we go to priority chart 2 from priority chart 1. If the numbering was completed then ok. But, if it was not done by above priority charts then we go to the last priority chart which was according to the evaluation of animals.

According to the above mentioned process we get the one-one correspondence from the different species to natural numbers. For our example of Fig. 3, the following order relation obtained as

1. Corn; 2. Grasshopper; 3. Rat; 4. Frog; 5. Snake; 6. Owl.

And the corresponding adjacency matrix was as follows.

Here, it is assumed that Grasshopper and Rat eat Corn [40%, 45%] and [25%, 30%] respectively and no other organisms eat Corn. And so for other organisms.

This is the adjacency matrix $A(\overrightarrow{G})$ of the food web $\overrightarrow{G}$ shown in Fig. 3. Note that all the entries for column 1 (corn) and row 6 (owl) are Ø, as 1 has no incident edges and 6 has no outgoing edges. Remove these column and row from the adjacency matrix

	1	2	3	4	5	6
1	[0, 0]	[0.4, 0.45]	[0.25, 0.3]	[0, 0]	[0, 0]	[0, 0]
2	[0, 0]	[0, 0]	[0.15, 0.2]	[0.2, 0.25]	[0, 0]	[0, 0]
3	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.3, 0.35]	[0.3, 0.35]
4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.35, 0.4]	[0, 0]
5	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.25, 0.3]
6	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]

$A(\overrightarrow{G})$ and let the new matrix be $\overline{A}(\overrightarrow{G})$ which is given by

$$\overline{A}(\overrightarrow{G}) = \begin{bmatrix} [0.4, 0.45] & [0.25, 0.3] & [0, 0] & [0, 0] & [0, 0] \\ [0, 0] & [0.15, 0.2] & [0.2, 0.25] & [0, 0] & [0, 0] \\ [0, 0] & [0, 0] & [0, 0] & [0.3, 0.35] & [0.3, 0.35] \\ [0, 0] & [0, 0] & [0, 0] & [0.35, 0.4] & [0, 0] \\ [0, 0] & [0, 0] & [0, 0] & [0, 0] & [0.25, 0.3] \end{bmatrix}$$

5.3 Measuring energy of the food web

The adjacency matrix $\overline{A}(\overrightarrow{G})$ of the food web $\overrightarrow{G}$ is an upper triangular matrix for this particular food web, but it is not true for other cases. Therefore, all diagonal elements of the matrix $\overline{A}(\overrightarrow{G})$ are eigenvalue corresponding to the eigenvectors $\kappa\,[\emptyset, \emptyset, \emptyset, \ldots, \emptyset, \ell, \emptyset, \ldots, \emptyset]^T$, where $\ell = [1, 1]$ be the unit element of the column vector and is the ith entries with $i \in \{1, 2, 3, 4, 5\}$ and $a_{ii} \leq \kappa$. Therefore, $[0.4, 0.45]$, $[0.15, 0.2]$, $[0, 0]$, $[0.35, 0.4]$, $[0.25, 0.3]$ are the eigenvalues of the adjacency matrix $\overline{A}(\overrightarrow{G})$ of the directed graph $\overrightarrow{G}$. So the spectrum of $\overline{A}(\overrightarrow{G})$ is

$$\sigma(A) = \{[0, 0],\ [0.15, 0.2],\ [0.25, 0.3],\ [0.35, 0.4],\ [0.4, 0.45]\}.$$

Thus, the energy of the graph (food web) $\overrightarrow{G}$ is

$$\begin{aligned} \mathbb{E}(\overrightarrow{G}) &= \sum_{i=1}^{5} [\underline{\xi}_i, \overline{\xi}_i],\ \text{since all eigenvalues are positive} \\ &= [0, 0] + [0.15, 0.2] + [0.25, 0.3] + [0.35, 0.4] + [0.4, 0.45] \\ &= [0 + 0.15 + 0.25 + 0.35 + 0.4, 0 + 0.2 + 0.3 + 0.4 + 0.45] \\ &= [1.15, 1.35]. \end{aligned}$$

The procedure to find energy of an ecology is stated below:
Input: A food web of an ecology.
Step 1: Construct an IVFG ($\overrightarrow{G}$) based on pre-predator relation.
Step 2: Compute the membership values of each edge based on the consumptions of food of the predators.
Step 3: Find the eigenvalues and eigenvectors of the IVFG.
Step 4: Find the energy $\mathbb{E}(\overrightarrow{G})$ of the IVFG $\overrightarrow{G}$.

Output: The energy $\mathbb{E}(\overrightarrow{G})$.

But, there is a big question, what is the significance of $\mathbb{E}(\overrightarrow{G})$? However, definitely the energy of a food web must be related with the stability of that particular food web in its ecosystem.

6 Conclusion

The eigenvalues and eigenvectors of a fuzzy graph are investigated by some authors without max–min operations and obtained any real numbers as eigenvalues. These eigenvalues are not acceptable as the fuzzy matrix theory is established over max–min operators and all members lie on [0, 1]. Some papers are published to find eigenvalues and eigenvectors when the elements of the matrix are triangular fuzzy numbers. But, as per our knowledge no papers are available to find eigenvalues and eigenvectors for IVFMs. In this paper, an attempt is made to find eigenvalues and eigenvectors of an IVFM and hence for an IVFG. Only the max–min operators are used to find such parameters of an IVFG. Obviously, execution of max–min operators on IVFM is a very difficult task. Also, the energy of an IVFG is evaluated. The very meaningful work of this article is the computation of energy of an ecological system. The consumption of food of the predators are considered as fuzzy intervals and constructed an IVFG. It may be noted that the consumption of food of predators cannot be measured certainly. So the fuzzy model is essential. From this IVFG, the energy of the ecological system is computed. The similar approach may be used to find the eigenvalues and eigenvectors and energy of any kind of ecosystem.

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Author Contributions All the authors contribute equally to this work.

Declaration

Conflict of interest The authors declare that they have no conflict of interest.

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