# 2022

# Mathematics

## [Honours]

## (B.Sc. First Semester End Examination-2022)

## PAPER-MTMH C101

## (Calculus, Geometry & History of Mathematics)

***Full Marks: 60*** ***Time: 03 Hrs***

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

*Illustrate the answers wherever necessary*

## Group-A

## [Calculus]

**1. Answer any two questions** $2 \times 2 = 4$

**(a)** Evaluate $\lim_{x \to \pi/2} (tanx)^{cosx}$

(b) Evaluate the area bounded by the parabola $y^2 = 4ax$ and any double ordinate of it,

say $x = x_1$.

(c) Find if there is any point of inflexion on the curve

$y - 3 = 6(x - 2)^5$

**2. Answer any two questions** $2\times 5=10$

(a) Prove that the eight points of intersection of the asymptotes of the curve $xy(x^2-y^2)+a(x^2+y^2)-a^3=0$ with the curve lie on a circle whose centre is at the origin

(b) Find the reduction formula of $\int \frac{\sin^m x}{\cos^n x}dx$ where m and n are positive integers greater than 1.

Hence find the value of $\int \frac{\sin^5 x}{\cos^6 x}dx$

(c) Let. $P_n = D^n(x^n \log x)$ Prove that the recurrence relation $P_n = n.P_{n-1}+(n-1)!$

Hence show that $P_n = n!\left(\log x+1+\frac{1}{2}+\frac{1}{3}..........+\frac{1}{n}\right)$

**3. Answer any one question** $1\times 10=10$

(a) (i) Determine the envelope of the circles described on the radii vectors of the curve $r^n = a^n cosn\theta$ as diameter.

(ii) Find the value of p and q such that

$$\lim_{x\to 0}\frac{x(1-pCosx)+qSinx}{x^3}=\frac{1}{3}$$

(iii) Proved that the curve $y = 2\sqrt{ax}$ is concave to the foot of the ordinate everywhere except at the origin. 4+4+2

(b) (i) Find the volume of the solid generated by revolving the Cardioide $r = a(1+\cos\theta)$ about the initial line.



(ii) Find the perimeter of the Cardioide $r = a(1-\cos\theta)$ and show that the arc of the upper half of the curve is bisected at $\theta = \frac{2\pi}{3}$.

## Group-B

## [Geometry]

**4. Answer any six question** $6\times 2=12$

(a)Find the point on the conic $\frac{l}{r}=1-\cos\theta$ which has the smallest radius vector

(b) A variable plane passes through a fixed point. Show that locus of the foot of the perpendicular from the origin to the plane is a sphere.

(c) Reduce the equation $5x^2$ $-2y^2-30x+8y$ to the form $ax^2+by^2=1$ by proper translation of axes without rotation.

(d) Find the value of k for which the plane $2x-2y+z+k=0$ touches the sphere $x^2+y^2+z^2-2x-4y-2z-3=0$

(e) If 2x=y=2z be one of the line of the set of three mutually perpendicular generators of the cone 11yz+6zx-14xy=0 then find the equations of the other two.

(f) Prove that radius of a circle do not alter due to rigid body motion.

(g) Find the equation of the cylinder generated by the straight lines parallel to the z-axis and passing through the curve of intersection of the plane lx+my+nz=p and the surface $x^2+y^2+z^2=1$

(h) Find the condition that the straight line $\frac{1}{r}=a\cos\theta+b\sin\theta$ may touch the circle $r=2k\cos\theta$

(i) Find the equation of the straight lines in which the plane $2x+y-z=0$ cuts the cone $4x^2-y^2+3z^2=0$

**5. Answer any one question** $1\times5=5$

(a) Reduce the equation $x^2-5xy+y^2+8x-20y+15$ to its canonical form and determine the type of the conic represented by it.

(b) Show that the perpendiculars from the origin to the generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ lie upon the cone $\frac{a^2(b^2+c^2)^2}{x^2}+\frac{b^2(a^2+c^2)^2}{y^2}=\frac{c^2(a^2-b^2)^2}{z^2}$

**6. Answer any one question** $1\times10=10$

(a)(i)Prove that the locus of a point for which three mutually perpendicular tangent lines can be drawn to the paraboloid $ax^2+by^2+2z=0$ is $ab\left(x^2+y^2\right)+2(a+b)z=1$

(ii) If P and Q be the variable points on the conic $\frac{1}{r}=1-e\cos\theta$ with vertical angle $\alpha$ *and* $\beta$ where $\alpha-\beta=2\gamma=$ constant, then show that the chord PQ touches the conic $\frac{1}{r}\cos\gamma=1-e\cos\gamma\cos\theta$ and that this conic has the same directrix as the original one. 5+5



(b) (i) Two spheres of radii $r_1$ and $r_2$ cut orthogonally. Prove that radius of their common circle is $\frac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$

(ii) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the co-ordinate areas at A, B, C. Find the equation of the cone generated by the straight lines drawn from O to meet the circle ABC 5+5

## Group-C

## [History of Mathematics]

**7. Answer any two questions** $2\times2=4$

(a) What is Sulbasutra? State Katyayana Sulbasutra.

(b) Write some propositions of Thales in Geometry.

(c) What was Euclid Contribution to mathematics?.

**8. Answer any one question** $5\times1=5$

**(a)** Write down the contribution of Pythagorasin Geometry and music related to mathematics. 3+2

(b) Discuss Euclidean constructions, the classical Problems, and their role in the history of Greek –mathematics, Discuss Euclid's elements. 3+2