2022

Mathematics

[Honours]

(B.Sc. First Semester End Examination-2022)

PAPER-MTMH C102

(Classical Algebra, Abstract Algebra-I and Linear Algebra-I)

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Group-A

[Classical Algebra]

1. Answer any two questions: $2\times 4=8$

a) Prove that $|z_1+z_2|\leq |z_1|+|z_2|$ where z_1 and z_2 are two complex numbers.

b) If α,β,γ be the roots of the equation $x^3-px^2+qx-r=0$ then find the value of $\sum\alpha^2\beta^2$

c) If a,b,c are positive, prove that $\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}\geq\frac{3}{2}$.

d) If the signs of a polynomial equation be all positive, show that it can not have positive root.

e) Find the condition that sum of two roots of the equation $x^3+bx+c=0$ is zero.

f) Find all values of $(-i)^{\frac{3}{4}}$.

2. Answer any one question: $1\times5=5$

(a) Solve the equation using Cardan's method $x^3+3x^2+6x+4=0$.

(b) If a_1, b_2..... a_n be all positive real numbers and $S=a_1+a_2+.....+a_n$

prove that $\frac{S}{S-a_1}+\frac{S}{S-a_2}+.......+\frac{S}{S-a_n}\geq\frac{n^2}{n-1}$

3. Answer any one question: $1\times10=10$

a) i). Reduce the equation $4x^4-85x^3+357x^2-340x+64=0$ to a reciprocal equation and then solve it.

ii) If $\alpha=\cos\frac{2r\pi}{n}+i\sin\frac{2r\pi}{n}$ and if r and p are prime to n, then prove that $1+\alpha^p+\alpha^{2p}+\alpha^{3p}\ldots\ldots\ldots.\alpha^{(n-1)p}=0$.

(b) (i) If α be an imaginary root of the equation $x^7-1=0$, find the equation whose roots are $\alpha+\alpha^6,\alpha^2+\alpha^5,\ \alpha^3+\alpha^4$

(ii) If x,y,z are positive real numbers and $x+y+z=1$ prove that

$$8xyz\leq(1-x)(1-y)(1-z)\leq\frac{8}{27}$$

Group-B

[Abstract Algebra-I]

4. Answer any two questions: $2\times2=4$

(a) Let $f:R\to R$ is defined by $f(x)=|x|+x,\ x\in R$ and $g:R\to R$ is defined by $g(x)=|x|-x,\ x\in R$. Find fog and gof.

b) Find the general solution in integers of the equation $5x+12y=80$.

(c) If a is prime to b then show that a^2 is prime to b^2.

5. Answer any two questions: $5\times2=10$

a) If p be a prime show that $\sqrt{p}$ is not a rational number.

b) Let $S=\{x\in R:-1<x<1\}$ and $f:R\to S$ be defined by $f(x)=\frac{x}{1+|x|},x\in R$. Show that f is bijection. Determine f^{-1}.

c) State division algorithm. Use division algorithm to prove that the square of an odd integer is of the form $(8k+1)$ where k is an integer.

Group-C

[Linear Algebra-I]

Group-C (Linear Algebra)

6. **Answer any four questions:** $4 \times 2 = 8$

a) If A is an orthogonal matrix then prove that $A^{-1} = A^T$.

b) Let V be a vector space over a field F, then $0\boldsymbol{\alpha} = \mathbf{0}$.

c) Find the dimension of the subspace S of $\mathbb{R}^3$ defined by $S = \{(x, y, z) : 2x + y - z = 0\}$.

d) Find the eigen value of the diagonal matrix.

e) Is the mapping $S: \mathbb{R}^3 \to \mathbb{R}$ defined by $S(x, y) = x^2$ linear? justify.

f) Is the set $S = \{(1, -2, -1), (3,0,1), (1,4,3)\}$ linearly dependent? Justify.

7. **Answer any one questions:** $1 \times 1 = 5$

a) State and prove caley - Hamilton theorem.

b) Define rank of the matrix. Determine the conditions for the system of equations has only one solution, many solutions, no solution: $x + y + z = 6, x + 2y + 3z = 10,\ x + 2y + az = b$.

8. **Answer any one questions:** $10 \times 1 = 10$

a) i) Diagonalise the matrix $\begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$

ii) If X and Y be two eigen vectors of a square matrix A corresponding to two distinct eigen values x and y respectively then prove that X and Y are linearly independent. 5+5

b) i) Obtain the fully reduced normal form of the matrix.

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{bmatrix}$$

Hence find the rank of the matrix.

ii) A linear mapping $S: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $S(x, y, z) = (x - y + 2z, x + 2y + z, x + y + 3z)$. Show that S is non- singular and determine S^{-1}. 5+5