M.Sc. First Semester End Examination, 2022

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-102

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

[COMPLEX ANALYSIS]

Answer question no. 1 and any four from the rest

1. Answer any four questions: $2\times4=8$

a) Is $f(z)=|z|^2$ analytic?

b) Determine the region of w-plane when the region bounded by $x=0, y=0, x=2, y=3$ in z -plane under the map $w=z\sqrt{2}e^{i\pi/4}$.

c) If $f(z)=\frac{z^2+5z+6}{z-2}$, does Cauchy's theorem apply when $|z|=3$?

d) Evaluate $\int_0^{2+i} \bar{z}^2 dz$ along real axis from $z = 0$ to $z = 2$ and then along a line parallel to $y-$axis fron $z = 2$ to $z = 2 - i$.

e) Expand $f(z) = \frac{1}{z}$ as a series about $z = 1$.

f) Find the points at which $w = \sin(z)$ is not conformal.

2. a) Find the Taylor or Laurent series expansion of the function $f(z) = z/((z-1)(z-3))$ for the region of convergence: $1 < |z| < 3$

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b) When $\log(z) = In(r) + i\theta,\ r = |z| > 0$, Show that $\log(i^2) \neq 2\log(i)$ for $\frac{3\pi}{4} < \theta < \frac{11\pi}{4}$, while $\log(i^2) = 2\log(i)$ for $\frac{\pi}{4} < \theta < \frac{9\pi}{4}$. 4 + 4

3. a) State and prove Morera's theorem.

b) The only singularities of a single valued function $f(z)$ are poles of order 1 and 2 at $z = -1$ and $z = -2$, with residues at these poles 1 and 2 respectively. If $f(0) = \frac{7}{4}, f(1) = 5/2$, determine $f(z)$. 4 + 4

4. a) Apply the calculus of residues to evaluate the integral

$$\int_0^{\infty} \frac{x^6}{(x^4 + a^4)^2} dx, a > 0.$$

b) Use Rouche's theorem determine the number of zeros of the polynomial $f(z) = z^{10} - 6z^7 + 3z^3 + 1$ in $|z| < 1$. 5 + 3

5. a) Find a conformal map of the unit disk $|z| < 1$ onto the right half-plane Re $(w) > 0$.

b) Find the Mobious transformation that maps 1, 0, -1 to the respective points i, ∞, 1. 6 + 2

6. a) If $f(z) = u + iv$ is an analytic function and $u - v = \frac{-\cos x + \sin x - e^y}{2\cosh y - \cos x}$, find $f(z)$ when $f(\pi) = 1/2$.

b) Find the value of $\int_0^{\infty} \frac{\cos x}{\sqrt{x}} dx$, by contour integration. 4 + 4

7. a) If a function $f(z)$ is analytic for all finite values of z and as $|z| \to \infty$ $|f(z)| = A|z|^k$, then prove that $f(z)$ is a polynomial of degree less and equal to k.

b) Evaluate $\int_C \frac{e^z}{z^2(z+1)^3} dz$ where $C: 9x^2 + 4y^2 = 36$ 4 + 4

[Internal Assessment – 10]
