M.Sc. First Semester End Examination, 2022

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-105

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

[CLASSICAL MECHANICS AND NON-LINEAR DYNAMICS]

Answer question no. 1 and any four from the rest

1. Answer any four questions: $2\times4=8$

(i) Define the Lagrangian and Hamiltonian of a dynamical system. Compare these two functions.

(ii) What do you mean by non-inertial frame? Give an example of a non-inertial frame.

(iii) Is poisson bracket commutative? Justify your answer.

(iv) Show that for conservative holonomic dynamical system,

$$\frac{\partial L}{\partial \dot{q}_j} = \int \frac{\partial L}{\partial q_j} dt.$$

(v) State basic postulates of special theory of relativity..

(vi) Show that the expression $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz's transformation.

2. Deduce Lagrange equations of motion for unconnected holonomic and conservative force. 8

3. (i) Deduce the Euler-Lagrange equation to obtain the curve $y = y(x)$ such that the functional $J = \int_{x_0}^{x_1} f(x, y, y')dx$ is stationary.

(ii) Obtain the curve for which the surface revolution is minimum 4+4=8

4. (i) The Hamiltonian of a dynamical system is given as

$$H = qp^2 - qp + bp$$

where b is a constant. Solve the problem.

(ii) Show that the Poisson bracket is invariant under the canonical transformation. 4+4=8

5. (i) If a body in the northern hemisphere falls freely to the ground from a height h, show that it strikes the ground at

$$\frac{2}{3}\omega h\left(\frac{2h}{g_e}\right)^{\frac{1}{2}}\cos\lambda$$

to the east, where ω is the earth's angular velocity, g_e is the acceleration due to the combined effect of gravity and centrifugal force and λ is the latitude of the place.

(ii) What is the effect of the Coriolis force on a particle falling freely under the action of gravity? 4+4

6. The principal moments of inertia of a body at the centre of mass are $A, 3A, 6A$. The body is so rotated that its angular velocities about the axes are $3n, 2n, n$ respectively. If in the subsequent motion under no force, $\omega_1, \omega_2, \omega_3$ denote the angular velocities about the principal axes at that time t, show that

$\omega_1 = 3\omega_3 = \frac{9n}{\sqrt{5}}\operatorname{sech} u$ and $\omega_2 = 3n\tanh u$ where

$u = 3nt + \frac{1}{2}log_e 5$ 8

7. (i) Consider the following nonlinear dynamical system,

$$\frac{dx}{dt} = x^2y - x^5, \frac{dy}{dt} = -y + x^2$$

Study the stability of the system at the origin.

(ii) Let, m_0 be the mass of a particle at rest and m be the mass of the same particle when it is moving with the velocity v. Then show that in relativistic mechanics$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where c is the velocity of light.

[Internal Assessment – 10]
