M.Sc. Third Semester End Examination, 2022

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-301

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Partial differential Equation and Generalized Functions]

Answer question no. 1 and any four from the rest

1. Answer anyfour questions: 4x2=8

   a) State the Basic existence theorem for Cauchy problem.

   b) Define 'Domain of dependence' of the one dimensional wave equation.

   c) If $L(u) = Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y$ then find its adjoint operatorL^*, where A, B, C, D, $E$and F are functions of x and y.

   d) Using the method of separation of variables, solve the equation

   $$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$$

   e) Describe 'Spherical mean' of a harmonic function.

f) Show that $\delta(-t) = \delta(t)$ where δ is the Dirac delta function.

2. a) Reduce the following equation to a canonical form and hence solve it. 6+2=8

$$yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$$

b) If a harmonic function vanishes everywhere on the boundary, then show that it is identically zero.

3. a) Let u be a harmonic function in a region $\mathbb{R}$. Also, let $P(x, y, z)$ be a given point in $\mathbb{R}$ and $S(P,r)$ be a sphere with centre at P of radius r such that is fully contained in the region $\mathbb{R}$. Then show that

$$u(P) = \bar{u}(r) = \frac{1}{4\pi r^2} \iint_{S(p,r)} u(Q)ds$$

b) Solve : $\left(x^2D^2 - 2xyDD' + y^2D'^2 - xD + 3yD'\right)u = 8\frac{y}{x}$

Symbols have their usual meaning. 4+4=8

4. a) Prove that the total energy of a string, which is fixed at the points x=0, u=l and executing small transverse vibration, is given by

$$\frac{1}{2}T\int_0^l \left[\left(\frac{\partial y}{\partial x}\right)^2 + \frac{1}{c^2}\left(\frac{\partial y}{\partial t}\right)^2\right]dx$$

Where $c2 = \frac{T}{\rho}$, ρ is the uniform linear density and T is the tension. Show, also that if $y = f(x - ct)$, $0 \le x \le l$ then the energy of the wave is equally divided between the potential and kinetic energy.

b) Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the following conditions:

i) $u(0,t) = u(2,t) = 0$

ii) $u(x,0) = \sin^3 \frac{\pi x}{2}$

iii) $u_t(x,0) = 0$ 5+3=8

5. A thin rectangular homogeneous thermally conducting plate lies in the xy-plane defined by $0 \le x \le a, 0 \le y \le b$. The edge $y = 0$ is held at the temperature $cx(x - a)$ where c is a constant, while the remaining edges are held at 0^0. The other faces are insulated and no internal sources and sinks are present. Then obtain the steady state temperature distribution inside the plate. 8

6. Obtain the solution of the one dimensional diffusion equation satisfying the following BCs :

a) T is bounded as $t \to \infty$

b) $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$, for all t

c) $\left.\frac{\partial T}{\partial x}\right|_{x=a} = 0$, for all t

d) $T(x, 0) = x(a - x)$, $0 < x < a$ 8

7. a) Find the derivative of the Heaviside unit step function.

   b) Establish the d'Alembert's formula of the Cauchy problem for the non-homogeneous wave equation. 2+6=8

Internal Assessment - 10
