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M.Sc. Third Semester End Examination, 2022

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-302

Full Marks: 50 *Time: 02 Hrs*

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Transform and Integral Equations]

Answer question no. 1 and any four from the rest

1. Answer anyfour questions: 4x2=8

a) Find $f(\alpha)$ if it exist for the function $F(p)=\dfrac{p+a}{p^2+b^2}$ where $F(p)$ is the Laplace transform of $f(x)$.

b) Define an integral equation with an example.

c) Deduce the initial value problem corres ponding to the integral equation

$$u(x)=x+\int_0^x (x-t)u(t)dt$$

d) What is the nature of the integral equation

$$f(x)=\int_a^b \frac{1}{x(t-a)}\phi(t)dt$$

e) Define the inversion Formula cosine transform of the function $f(x)$, what happens if $f(x)$ is continuous?

f) Define the wavelet function and analyse the parameters involving in it.

2. a) If the Fourier Transform $F(k)$ of a function $f(x)$ exists then $F(k)$ is a continuous function of k

4+4=8

b) (i) Solve the following ODE by Laplace transform technique:

$y''(t)+2y'(t)+5y(t)=h(t)$ with initial condition $y(0)=0$, and $y'(0)=0$ where $h(t)=\begin{cases}1, & 0<t<\pi \\ 0, & t>\pi\end{cases}$

3. i) Solve the integral following equation

$y(x)=f(x)+\lambda\int_{-1}^{1}\left(xt+x^2t^2\right)y(t)dt$ and find the eigen values

ii) Find the Laplac transform of $f(x)=[x]$ where $[x]$ represents the greatest integer less than or equal to x 6+2=8

4. a)If $L\{f(t)\}=F(p)$ Show that $L\left\{\frac{f(t)}{t}\right\}=\int_0^{\alpha}F(p)dp$ provided that $\lim_{t\to 0}\frac{f(t)}{t}$ exists

b) Find f(x) if its Fourier sine Transform is $\sqrt{\frac{2}{\pi}}\cdot\frac{k}{1+k^2}$ 5+3=8

5. a) Using residue theorem find $f(x)$ where Laplace transform

$$F(p)=\frac{p}{(p-2)(p^2+4)}$$ 4+4=8

b) Find the resolvent kernel and using this solve the integral equation $\phi(x)=x+\int_0^1(t-x)\phi(t)dt$

6. a) Find the eigen value and eigen functions of the integral equation $\phi(x)=\lambda\int_0^{2\pi}Sin(x+t)\phi(t)dt$

b) Find the solution of the equation $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$ $x>0,\ t>0$ which remains bounded for $x\geq 0$ and following initial and bounded conditions $u(x,0)=0,\ u(0,t)=f(t)$ 4+4= 8

7. State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that

$$\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)}=\frac{\pi}{ab(a+b)},a,b>0$$ 8

Internal Assessment - 10