M.Sc. Third Semester End Examination, 2022

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-305

Full Marks: 100 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

USE SEPARATE ANSWER SCRIPT FOR TWO UNITS

Unit – I

MTM-305A

[Dynamical Oceanology]

Full Marks - 50

Attempt any five questions: **$5 \times 8 = 40$**

1. Derive Gibb's-Duhem thermodynamically relation for sea water. Define adiabatic temperature gradient of sea water.

   Prove that $\left(\frac{\partial \rho}{\partial p}\right)_{\eta,s} = \Gamma \frac{\partial \rho}{\partial T} + \frac{\partial \rho}{\partial p}$ (symbols have their usual meanings).

2. Derive the necessary conditions of thermodynamic equilibrium of a finite volume of sea water.

3. Derive the boundary conditions at the free ocean surface $F(\vec{r},t)=0$. Express Brunt- Väisälä frequency in terms of C_p and C_v.
4. Derive the equation of motion of sea-water.
5. Derive the field equations approximately according to β-plane approximation.
6. Establish the equation of pure drift currents of sea water on a rotating earth.
7. Derive the Fridman's Equation for vorticity in terms of motion relative to the Earth.

Internal Assessment - 10

Unit – II
MTM-305B
[Advanced Optimization and Operations Research]
Full Marks - 50

Answer Question No. 1 and four from rest

1. Answer any four questions: $4\times2=8$

a) Write down the limitations of Fibonacci method.

b) What are the advantages of revised simplex method over simplex method?

c) Prove that f(X) increases at the fastest rate in the direction of $\nabla f(X)$

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d) Write the iterative scheme of Steepest Descent method.

(e) Find the conjugate directions or the symmetric matrix $\begin{pmatrix}4 & 5\\5 & 4\end{pmatrix}$

(f) What is unimodal function? Compare the analytical method and numerical methods for optimizable.

2. Solve the following LPP by revised simplex method

Max $z=6x_1-2x_2+3x_3$

Subject to $2x_1-x_2+2x_3\le 2$

$x_1+4x_3\le 4$

$x_1,x_2,x_3\ge 0$ 8

3. Using Fletcher and Reeves method, minimize $f(x_1,x_2)=x_1-x_2+2x_1^2+2x_1x_2+x_2^2$ starting from the point $\begin{bmatrix}0\\0\end{bmatrix}$ 8

4. What is golden ratio? Using golden section method

maximize $f(x)=\begin{cases}(x^2-6x+13)/4 & , x\le 4\\ x-2, & x>4\end{cases}$ in the interval [2,5]

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2+6

5. Prove that if quadratic function $Q(X)=1/2\ X^TAX+B^TX+C$ where A be $n\times n$ symmetric matrix, $B,X\in\Re^n$ and C is real constant is minimized sequentially once along each direction of a set of n A-conjugate directions then the global minimum of Q(X) will bw located at a before the nth step regardless of the starting point and the order n which the directions are used.

6. Using Cutting plane method

Max $f(x_1,x_2)= 7-2x_1-4x_2$

subject to $(x_1-4)^2+2(x_2-3)^2 \leq 12$

$x_1+2x_2 \leq 6$

$1 \leq x_1 \leq 6$

$1 \leq x_2 \leq 6$ 8

7. What is the usefulness of post optimality analysis?

Find the ranges of the discrete change of cost vector of LPP

Maximize Z=cx

subject to Ax=b, x>=0 so that the condition of optimality remains unchanged. 2+6

Internal Assessment - 10