2021

Mathematics

[Generic]

(CBCS)

(B.Sc. FirstSemester End Examinations-2021)

MTMH-GE-101

[*Numerical Methods and Differential Calculus – I*]

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Group – A
[*Numerical Methods*]

1. Answer any SEVEN questions: 7x2=14

a) Why relative error is better indicator of the accuracy of a computation than absolute error ?

b) Prove that $\Delta\nabla f(x) = \Delta f(x) - \nabla f(x)$ where Δ and ∇ are forwarded backward operator respectively.

c) Prove that third order difference of $f(x) = x^2 + 2x + 1$ is zero

d) Define relative error and percentage error with an example.

e) Write down the sufficient condition for the convergence of the Gauss-Seidel iteration method.

f) Why iteration methods is called as fixed point iteration ?

g) Show that the sum of Lagrangian functions is 1.

h) Show that Simpson's $\frac{1}{3}$ rule is exact for a polynomial of degree 3.

i) Why Newton's Raphson method is called method of tangent?

j) Define degree of precision for numerical integration. What is the degree of precision of Simpson's $\frac{1}{3}$ rule?

k) Find the missing term in the following table.

x :	0	1	2	3	4
$f(x)$:	1	3	9	-	81

2. Answer any twoquestions **5x2=10**

a) Describe the Newton's Raphson methods for computing a simple root of an equation $f(x)=0$ What is the sufficient condition for Newton's Raphson method ?

b) Solvethe system of equation by Gauss Seidel method correct upto three significant figures.

$$3x+y+z=3$$
$$2x+y+5z=5$$
$$x+4y+z=2$$

c) (i) Find the percentage error in $f(x)=2x^3-4x$ at $x=1$ when error in x is 0.04.

(ii) Find the interpolating polynomial using any method from the following table

x	0	1	2	3	4	5
y	-3	-5	-11	-15	-11	7

3. Answer any ONE question **10x1=10**

a) i) Establish the general quadrature formula based on Newton's forward interpolation formula. Hence deduce Simpson's $\frac{1}{3}$ rule. 4+1

ii) Evaluate $\int_0^1 \frac{xdx}{1+x}$ by Trapezoidal rule, taking six intervals. Calculate absolute error if occurs. 4+1

b) i) Given $\frac{dy}{dx}=y^2-x^2, y(0)=2$. Find $y(0.1)$ and $y(0.2)$ by second order RungeKuttamethod.and by second order Runge –Kutta method. 5

ii) Determine the largest eigen value and corresponding eigen vector for the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ 5

Group – B
[*Differential Calculus - I*]

4. Answer any THREE questions 2x3=6

a) Find the reduction formula for $\int Sec^n x dx$,n being positive integer >1.

b) Find $\lim_{x \to 0} \frac{x - Sinx\, Cosx}{x^3}$

c) If $y = \log(ax+b)$ then find y_n.

d) Find the points of inflexion on the curve $x = (\log y)^3$

e) Find the length of the circumference of a circle of radius a.

5. Answer any TWO questions 5x2=10

a) State Leibnitz's rule for nth order differentiation. If $y = \cos(m\, Sin^{-1} x)$ then show that 4+1

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

b) Find the volume of the solid generated by revolving the cardiode $r = a(1-\cos\theta)$, about the initial line. 5

c) Evaluate the reduction formula for $\int \tan^n x\, dx$ n being positive integer > 1. Hence find $\int_0^{\pi/4} \tan^4 x dx$. 4+1

6. Answer any ONE question 1x10=10

a) (i) Find the asymptotes of the cubic

$$x^3 - 2y^3 + xy(2x-y) + y(x-y) + 1 = 0$$

(ii) Find the value of p and q such that $\lim_{x \to 0} \frac{x(1-p\cos x) + qSinx}{x^3} = \frac{1}{3}$ 5

b) (i) Find the envelope of the curve $y = mx + \frac{a}{m}$, m being a parameter and a is a constant.

(ii) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \;\; Sinx\, dx$ where m, $n$are positive integer, show that

$$\mathrm{I}_{m,m} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \ldots + \frac{2^m}{m}\right]$$ 5

[The End]