Mathematics

[HONOURS]

(CBCS)

(B.Sc. Third Semester End Examinations-2023)

[Regular and Supplementary Paper]

MTMH-C301

Ordinary Differential Equations & Applications of Dynamics

[Use separate answer scripts for each group]

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Group – A

[Originary differential equations: Marks-36]

1. **Answer any THREE questions:** **3x2=6**

   a) Find the particular integral of $(D^3-2D+4)y=e^x cosx$

   b) Solve $\frac{dx}{x(y^2-z^2)}=\frac{dy}{y(z^2-x^2)}=\frac{dz}{z(x^2-y^2)}$

   c) Find the integrating factor of the differential equation
   $(2x^2y^2+y)dx-(x^3y-3x)dy=0$

d) The differential equation $\left(2x^2+by^2\right)dx+cxydy=0$ is made exact by multiplying the integrating factorx^{-2}. Find the relation between b and c.

e) Let $y(x)$ be a continuous solution of the $IVP:\frac{dy}{dx}+2y=f(x)$, , $y(0)=0$, where $f(x)=\begin{cases}1:0\le x\le 1\\ 0:x>1\end{cases}$. Then find $y(3/2)$

f) Let $y(x)=u(x)\sin x+v(x)Cosx$ be the solution of the differential equation $\frac{d^2y}{dx^2}+1=Secx$. Then find u(x)

2. Answer any FOUR questions **4x5=20**

a) Solve the differential equation

$$(2+x)^2\frac{d^2y}{dx^2}+(2+x)\frac{dy}{dx}+4y=2\sin\{2\log(2+x)\}$$

b) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\frac{e^{-x}}{x^2}$ by the method of variation of parameters.

c) Solve the following system of differential equations

$$\frac{dx}{dt}+\frac{dy}{dt}-2y=2\cos t-7\sin t$$

$$\frac{dx}{dt}+\frac{dy}{dt}+2x=4\cos t-3\sin t$$

d) Solve : $\frac{d^2y}{dx^2}+\frac{3dy}{dx}+2y=e^{e^x}$

e) If $y_1(x)=x$ is a solution to the $ODE:\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ then show that its general solution is $y(x)=c_1x+c_2\left[\frac{x}{2}\ln\left|\frac{1+x}{1-x}\right|-1\right]$

f) Solve: $\frac{dx}{x}=\frac{dy}{-2}=\frac{dz}{3x^2Sin(y+2x)}$

3. Answer any ONE question **1x10=10**

a) Given system of differential equation

$$\frac{dx}{dt}=-3x+4y$$

$$\frac{dy}{dt}=-2x+3y$$

i) Find the general solution of the system

ii) Find the critical point and state the nature of the critical point.

iii) Justify the stability and unstability of the critical point.

iv) Draw the phase portrait of the system. 4+2+2+2

b) i) Show that the point of infinity is a regular signature point of the equation $x^2y''+(3x-1)y'+3y=0$ 7+3

ii) Find the particular integral of $\left(D^2-4D+4\right)y=3x^2e^{2x}\,Sin2x$ 4

Group – B
[*Application of Dynamics: Marks - 24*]

1. Answer any TWO questions **2x2=4**

a) Explain the term "terminal velocity".

b) The law of motion in a straight line is $s = \frac{1}{2}vt$. Show that the acceleration is constant.

c) State Kepler's laws of planetary motion.

2. Answer any TWO questions **5x2=10**

a) Find the law of force to the pole when the path is $r = a(1-\cos\theta)$ and prove that if F be the force at the apse and v the velocity there, then $3v^2 = 4aF$

b) A spherical raindrop, falling freely, receives in each instant an increase of volume equal to k times its surface at that instant; find the velocity at the end of time t, and the distance fallen through in that time..

c) A particle, of mass m, is projected vertically under the gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{V^2}{g}[\lambda - log(1+\lambda)]$, where V is the terminal velocity and λVis the initial velocity.

3. Answer any ONE question **1x10=10**

a) State the principle of energy. Over a small smooth pulley is placed a uniform flexible cord; the latter is initially at rest and lengths $l + c$ and $l - c$ hang down on the two sides. The pulley is now made to move with constant upward acceleration f. Show that the string will have the pulley after a time $\sqrt{\frac{l}{f+g}} cosh\frac{l}{c}$. 2+8

b) i) A particle moves under a force $m\mu\{3au^4 - 2(a^2-b^2)u^5\}$, $a>b$, and is projected from an apse at a distance $a+b$ with velocity $\frac{\sqrt{\mu}}{a+b}$; show that the orbit is $r = a + bCos\theta$.

ii) Find the radial and cross-radial components of acceleration of a particle moving along a plane curve. 6+4

[The End]