## 2021

# Mathematics

## [HONOURS]

## (CBCS)

## (B.Sc. First Semester End Examinations-2021)

### MTMH-C101

### *Calculus , Geometry & History of Mathematics*

***Full Marks: 60***                                    ***Time: 03 Hrs***

*The figures in the right hand margin indicate marks*
*Candidates are required to give their answers in their own words as*
*far as practicable*
*Illustrate the answers wherever necessary*

### Group – A
### [*Calculus*]

1. **Answer any TWO questions:**                     **2x2=4**

   a) If the function $f(x) = ax^3 + bx^2$ has a point of inflexion at $(1, 2)$ then find the values of $a$ and $b$.

   b) If $I_n = \int_0^{\pi/4} \tan^n x \, dx$ show that $I_{n+1} + I_{n-1} = \dfrac{1}{n}$, $n$ being positive integer>1.

c) If $\displaystyle\lim_{x \to 0} \frac{Sin\,2x + aSinx}{x^3}$ be finite then find the value of $a$ and the limit.

d) Find the volume generated by the revolution about $x$ – axis of the area bounded by the loop of the curve $y^2 = x^2(2-x)$.

2. **Answer any two questions**           **2x5=10**

a) If $I_n = \displaystyle\int_0^{\pi/2} \cos^{n-2} x \, Sin \, nx \, dx$

show that $2(n-1)I_n = 1 + (n-2)I_{n-1}$ and hence deduce

$$I_n = \frac{1}{n-1} \qquad\qquad 3+2$$

b) State Leibnitz rule for nth derivative. Show that

$$\frac{d^n}{dx^n}\left(\frac{x^n}{1+x^2}\right) = n! \, Siny \left\{ Siny - {}^nc_1 \cos y \, Sin2y + {}^nc_2 \, \cos 2y \, Sin3y - \ldots \right\}$$

$$1+4$$

c) What do you mean by rectilinear asymptote? Find the asymptotes of the curve $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0$

3. **Answer any ONE question**           **1x10=10**

a) i) Prove that the length of the loop of the curve $x = t^2$, $y = t - t^3/3$ is $4\sqrt{3}$

ii) Find the envelope of the straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ where parameters a and b are connected by $a^2 + b^2 = c^2$.

b) i) Find the range of values of $x$ for which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards or downwards. Find also the points of inflexion.      2+2+1

ii) Find the area about the x-axis included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$

**Group – B**

**[*Geometry*]**

4. **Answer any SIX questions**           **6x2=12**

a) Show that the locus of point whose distance from the pole is equal to its distance from the straight line $r \cos\theta + k = 0$ is

$$2r \, Sin^2 \theta/2 = k$$

b) For what values of $a$ and $f$ so that the equation $ax^2 - 20xy + 25y^2 - 14x + 2fy - 15 = 0$ represents a conic without any centre ?

c) What angle the axes be turned to remove the term $xy$ from $x^2 + 2\sqrt{3} \, xy - y^2 = 4$ ?

d) Find the radius of the circle $3x^2 + 3y^2 + 3z^2 + x - 5y - 2 = 0$, $x + y = 2$

e) Show that the straight line $\dfrac{x-2}{1}=\dfrac{y}{-2}=\dfrac{x-4}{1}$ touches the

sphere $x^2+y^2+z^2-\dfrac{2}{3}x-y-\dfrac{4}{3}z=\dfrac{22}{3}$ at (1, 2, 3).

f) Show that the straight line $\dfrac{x-2}{2}=\dfrac{y-3}{-6}=\dfrac{z-1}{1}$ meets the

meets the conicoid $\dfrac{x^2}{16}+\dfrac{y^2}{9}-\dfrac{z^2}{4}=1$

g) Find the equation of cone with vertex (1, 2, 3) and the

guiding curve is $y^2=4ax,\ z=0$

h) Find the equation of cone which passes through the co-

ordinates axes as well as two lines $\dfrac{x}{1}=\dfrac{y}{-2}=\dfrac{z}{3}$ and

$\dfrac{x}{3}=\dfrac{y}{-1}=\dfrac{z}{1}$

i) Find the equation of cylinder whose generators are parallel

to x-axis and which passes through the curve of intersection

of the plane $2x-3y+z=2$ and the surface $3y^2-5z^2=10x$

**5. Answer any ONE questions**        **1x5=5**

a) Obtain the focus of the parabola $\sqrt{ax}+\sqrt{by}=1$.

If its axis passes through a fixed point, show that the locus

of the focus is a rectangular hyperbola.

b) Find the equation of the sphere which passes through the

points (2, 0, 0), (0, 2, 0) and (0, 0, 2) and has a least possible

radius.

**6. Answer any ONE question**        **1x10=10**

a) (i) A variable sphere passes through the points $(0,0,\pm c)$ and

cuts the straight lines

$y=x+and,$

$z=cj$

$y=-x+and,$

$z=-c$

at the points $P$ and $P'$. If $PP'=2a$, a constant, then show

that the centre of the sphere lies on the circle

$$z=0,\ x^2+y^2=\left(a^2-c^2\right)\cos ec^2 2\alpha$$

(ii) Prove that the conditions that the line of section of the

plane      $lx+my+nz=0$      and      the      cones

$ax^2+by^2+cz^2=0,\ fyz+gzx+hxy=0$ may be coincident,

are $\dfrac{bn^2+cm^2}{fmn}=\dfrac{cl^2+an^2}{gnl}=\dfrac{am^2+bl^2}{hlm}$      6+4

b) (i) Find the director sphere of the conicoid $ax^2+by^2+cz^2=1$

(ii) Show that six normal &can be drawn from any point

$P(\alpha,\beta,v)$ to the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$

**Group – C**
**[*History of Mathematics*]**

7. **Answer any TWO question**                 **2x2=4**

   a) Write the names of two Italian mathematics in the 17$^{th}$ century.

   b) How did Pythagoras relate mathematies to music?

   c) State BaudhayanaSulbasutra and KatyayanaSulbasutra.

8. **Answer any ONE question**                 **5x1=5**

   a) Write down the names and contributions of some Mathematicians in 17$^{th}$ Century.

   b) Write some contributions of Brahmagupta in Indian Mathematics.

**[The End]**