2023

B.Sc. (Honours)

B.Sc. Third Semester End Examination - 2023

PHYSICS

PAPER - CC5T

Full Marks : 40 *Time : 2 hours*

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

Group - A

1. Answer any five questions **5×2=10**

(a) What is Dirichlet's condition? 2

(b) Express, $f(x) = 1+x+x^2+x^3+x^4$ in terms of Legendre polynomial.

(c) Show that $\beta(m,n) = \frac{n-1}{m+n-1}\beta(m,n-1)$

(d) Show that $\Gamma(n) = \int_0^1 \left[\ln\left(\frac{1}{y}\right)\right]^{n-1} dy$, (n>0)

(Turn Over)

(e) Evaluate $\int_{-1}^{+1}[p_3(x)]^2\,dx$

(f) Express $8x^3+8x^2-6x+2$ in terms of Hermite polynomials.

(g) Find the value of $p_5(x)$ from Rodrigues formula.

(h) Can you expand tanx in fourier series? Justify your answer.

Show that, $H''_n(x) - 2x\,H'_n(x) + 2n\,H_n(x)=0$

Group - B

Answer any four of the follwing questions. **4×5=20**

2. Find the point on the plane $ax+by+cz=\beta$ at which the function $f(x)=x^2+y^2+z^2$ has a minimum value and find the minimum f. 5

3. Find Founer series for a fuction, $f(x)=x(1+x)$, $\pi<x<\pi$. Hence show that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^3}+\ldots\ldots\ldots\infty$

4. Using the expression of Bessel's function,

$$J_n(x)=\sum_{r=0}^{\infty}\frac{(-1)^r}{r!\,\Gamma(m+r+1)}\left(\frac{x}{2}\right)^{n+2r}$$

Show that

(i) $J_{m+1}(x)+J_{m-1}(x)=\frac{2m}{x}J_m(x)$

(ii) $J_{m-1}(x) - J_{m+1}(x) = 2J'_m(x)$ 3+2

5. (i) Derive the complex form of Fourier Series.

(ii) Deduction using the following generating function for $P_n(x)$:-

(a) $P_n(1)=1$ (b) $P_n(-1)=(-1)^n$ 2+2+1

6. Prove that

$$\int_0^{\infty}x^2e^{-x^4}dx.\int_0^{\infty}e^{-x^4}dx=\frac{\pi}{8\sqrt{2}}$$ 5

7. Prove that $H_n(-x)=(-1)^n\,H_n(x)$ 5

Group - C

Answer any one of the following : **1×10=10**

8. (a) Express the function

$$f(x)=\begin{cases}0 & -1<x<0\\ x & 0<x<1\end{cases}$$

in Fourier-Legandre expansion

(b) Prove that

$P'_{n+1}(x) + P'_{n-1}(x) = 2x\,P'_n(x) + P_n(x)$ 5+5=10

9. (a) A thin circular plate, with faces impervious to heat flow and the edge kept at Zero degree, has its initial temperature at t=0 as a function of F(r) of the radial

distance r. Find the subsequent temperature distribution in the plate.

(b) Show that

$$\beta(m,n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\, d\theta$$

(c) Find the relation between Beta and Gamma functions.

5+2+3