2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. ThirdEnd SemesterExaminations-2021)

MTMH-C302

Graph theory – II & Linear Algebra - II

Full Marks: 60 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Group – A
[*Graph – Theory - II*]

1. **Answer any TWO questions:** **2x2=4**
   a) Prove that the centre $Z(S_3)$ is normal in S_3, where S_3 is symmetric group.
   b) Let, G =$(Z_6,+), H = \{\overline{0}, \overline{3}\}$ .Then obtain all the elements of the quotient group G/H
   c) Let, G be a group of order 8 and x be an element of G of order 4. Show that . $x^2 \in Z(G)$

2. Answer any TWO questions **5x2=10**

a) Find all the homomorphism from the group (z_{10},+) to (z_{15},+)

b) Let, G be a group in which $(ab)^3 = a^3b^3$ for all $a, b \in G$.

.Prove that H$=\{x^3 : x \in G\}$ is a normal subgroup of G

c) Find the number of elements of order 5 in the group

$Z_{15}xZ_{10}$.

3. Answer any ONE question **10x1=10**

a) i) state and prove Cauchy's theorem for finite abelian group.

ii) G is a multiplicative group of order 8. Prove that $\phi : G \to G$ defined by $\phi(x) = x^3, x \in G$ is an omorphism.

6+4

b) i) Let, G_1, G_2 be two groups and Z(G_1), Z(G_2) be their respective centres. Prove that Z(G_1) X Z(G_2) is the centre of the group $G_1 \times G_2$

ii) Examine if the mapping is a homomorphism : G=S_3 and $\phi : G \to G$ is defined by $\phi(x) = x^2, x \in S_3$

iii) Show that an abelian group of order 15 is cyclic. 6+2+2

Group – B

[*Linear Algebra - II*]

1. Answer any EIGHT questions **8x2=16**

a) Give the definition of linear dependence and linear independence.

b) What do you mean by basic of a vector space.

c) What do you mean by nullity and rank of a linear mapping ?

d) Let V and W be vector space over a Field F and $T : V \to W$ be a linear map then prove that $T(\theta) = \theta'$ where θ and θ' are zero element in V and W resp.

e) The mapping $T : \Re^3 \to \Re^3$ defined by $T(x_1, x_2\, x_3) = (x_1 + 1,\ x_2 + 1,\ x_3 + 1), (x_1, x_2, x_3) \in \Re^3$ Examine that T is linear or not.

f) Give the definition and example of linear sum of two subspaces.

g) Let S be the subset of $\Re$ defined by $S = \{(x, y, z) \in \Re^3 : y = Z = 0\}$. then prove that S is a subspace of $\Re^3$

h) What do you mean by composition of linear mapping ?

i) Let V be a vector space over a field F, and U and W be two subspaces. Then which subspace is the smallent subspace of V containing the subspaces U and W and why ?

j) Find the condition on x, y so that the set of vectors is linearly dependent in $\Re^3 \{(x, y, 1), (y, 1, x), (1, x, y)\}$

k) Let S and T be linear mapping of $\Re^3$ to $\Re^3$ defined by $s(x, y, z) = (z, y, x), (x, y, z) \in \Re^3$ and $T(x, y, z) = (x + y + z, y + z, z), (x, y, z) \in$ □^{3} determine TS and ST

l) Give an example of a linear operator T on a vector space V such that $KerT = ImT$

2. Answer any TWO questions **5x2=10**

a) Prove that the subset D[a, b] of all real values differentiable function defined on [a, b] is a subspace of c[a, b]

b) D and T are linear mapping on the real vector space P_4 defined by $D\left(p(x)\right) = \frac{d}{dx} p(x), p(x) \in P_4$

$$T\left(p(x)\right) = x\frac{d}{dx} p(x), p(x) \in P_4$$

Relative to the basis $(1, x, x^2, x^3)$ of P_4, determine the matrix of each of the linear mappings D and T.

c) Find the dimension of the subspace $S \cap T$, where S and T are subspace of the vector space $\Re^4$ given by

$S = \left\{(x, y, z, w) \in \Re^4 : x + y + z + w = 0\right\}$

$T = \left\{(x, y, z, w) \in \Re^4 : 2x + y - z + w = 0\right\}$

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3. Answer any ONE question **1x10=10**

a) i) If $\left\{\alpha_1, \alpha_2, ..., \alpha_n\right\}$ be a basis of a finite dimensional vector space V over a field F, then prove that any linearly independent set of vectors in V contains at most n vectors

ii) Two subspaces of $\Re^3$ are $U = \left\{(x, y, z) : x + y + z = 0\right\}$ and $W = \left\{(x, y, z) : x + 2y - z = 0\right\}$. Find $\dim U, \dim U \cap W$ 7+3

b) $T : \Re^3 \to \Re^3$ be a linear map, find $KerT$ and $\operatorname{Im} T$ when T maps the basis vector

i) (1,0,0), (0,1,0), (0,0,1) of $\Re^3$ to vector (0,1,0), (0,0,1), (1,0,0)

ii) (0,1,1), (1,0,1), (1,1,0) of $\Re^3$ to vector (2,1,1), (1,2,1), (1,1,2) respectively. 5+5

[The End]