Mathematics

[HONOURS]

(CBCS)

(B.Sc. ThirdEnd SemesterExaminations-2023)

[Regular and Supplementary Paper]

MTMH-C303

Full Marks: 60 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[*REAL ANALYSIS - II*]

1. Answer any TEN questions: **10x2=20**

a) Find the value of $\lim_{x\to 0}|\operatorname{sgn}(x)|$ exists.

b) Show that $\lim_{x\to 0} f(x)$ does not exists where $f(x) = e^{1/3}$

c) Define the limit $\lim_{x\to 0} f(x) = -\alpha$

d) A function $f:\mathbb{R}\to\mathbb{R}$ satisfies $f(x+y) = f(x).f(y)$ for all $x, y \in \mathbb{R}$ and f is continuous at 0. Prove that f is continuous at every $C \in \mathbb{R}$.

e) Let $f(x) = \text{sgn}(x)\left|Sin\frac{1}{x}\right|, x \neq 0 = 0, x = 0$. Find the nature of discontinuity Φ of $f(x)$ at 0.

f) Let $f:[a,b] \to [a,b]$ be continuous in $[a,b]$ show that for some $\xi \in [a,b]$ $s.t$ $f(\xi) = \xi$

g) Show that $3^{Sinx} - 4\,Sinx$ has a solution in $\left(0, \frac{\pi}{2}\right)$

h) Prove that $f(x) = x^2, x \in \mathbb{R}$ is not uniformly continuous on $\mathbb{R}$

i) If $f:\mathbb{R} \to \mathbb{R}$ is differentials at $C \in \mathbb{R}$. Show that

$$f'(c) = \lim_{n \to \alpha}\left\{\frac{n}{c}\left\{f\left(1+\frac{1}{n}\right)\right\}\right\}$$

Where f satisfies $f(x.y) = f(x) + f(y)$

j) State Rolle's theorem for polynomial

k) State Taylor's theorem for $f:[a, a+b] \to \mathbb{R}$ with Langrange form of remainder.

l) Verify Language Mean Theorem for the function $f(x) = 2-(4-x)^{2/3}$ on $[2,5]$

m) Discuss the continuity of the function

$$f(x,y) = \begin{cases} \frac{|xy|}{\sqrt{x^2+y^2}} & (x,y) \neq 0 \\ = 0 & (x,y) = (0,0) \end{cases}$$

n) Let f, g be continuity form $\mathbb{R}$ to $\mathbb{R}$ and suppose that $f(r) = g(r)$ for all rational numbers. Is it true that $f(x) = g(x)$ for all $x \in \mathbb{R}$.

o) Find $f_x(0,0), f_y(0,0)$ of the function $f(x,y) = \log\log(1+xy)$

2. Answer any FOURquestions **5x4=20**

a) Let $I = (a,b)$ be a bounded open internal and $f: I \to \mathbb{R}$ be a monotone increasing on I proof that if f is bounded above on I then $\lim_{x\to b} f(x) = \underset{x \in (a,b)}{Sup} f(x)$

b) Let $f:(a,b) \to \mathbb{R}$ and f is continuous at $x = c$. If f is monotonic increasing in (a,c) and monotonic decreasing in (c,d) then proof that $f(x) = 0$ has a unique solution at $x = c$

c) The functions u and v are continuous and devisable on $\mathbb{R}$. If $uv' + vu' \neq 0 \forall (\alpha, \beta)$ then v is stanchly monotonic on (α, β) where α and β are two consecutive root of u.

d) Let $I = [a,b]$ and a function $f: I \to \mathbb{R}$ be differentiable on I. Let $f'(a) \neq f'(b)$. If K be a real number lying between $f'(a)$ and $f'(b)$ then there exists a point C in (a,b) such that $f'(c) = k$

e) A function f is defined on (-1, 1) by

$$f(x) = x^{\alpha} Sin\frac{1}{x^{\beta}} \; x \neq 0$$
$$= 0 \; x = 0$$

i) State condition on such that $f(x)$ is continuous at $x=0$

ii) For what value of β, $f(x)$ is continuous at $x=0$ when $\alpha = 0$

iii) Show that if $\alpha = 1$, $\beta > 0$ $f'(0)$ does not exists but $\alpha > 1$, $\beta > 1$ $f'(0)$ exists

f) Let $f(x,y) = \begin{cases} \frac{x^2 y(x-y)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$ verify they are equal or not. If not, then specify the reason

3. Answer any TWO question **10x2=20**

a) i) Let. $f:[a,b] \to$ □ State and prove Lagrange. Mean value theorem. Hence prove that if f has continuous derivative on $[a,b]$ then f is a Lipsachity function on $[a,b]$.

ii) Find local ϕ extrema, local extrema values, Saddle point if exists of the function

$$f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$$