M.Sc. First Semester End Examination, 2023
(Regular & Supplementary Paper)
Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-102
[COMPLEX ANALYSIS]

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate mark.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary

Answer question no. 1 and any four from the rest

1. Attempt any **four** questions: $2 \times 4 = 8$
   a) Is the function $f(z) = z|z|$ analytic?
   b) Under what condition/s the bilinear transformation $f(z) = \frac{az+b}{cz+d}$ have only one fixed point? Justify your answer.
   c) Evaluate $\int_0^{1+i} \bar{z}\, dz$ along the line from $z = 0$ to $z = 1 + i$.
   d) Evaluate: $\frac{1}{2\pi i}\int_C \frac{e^{-z}}{z-2} dz$ where C is circle $|z| = 3$.
   e) Define branch and branch cut for a multi-valued function f(z).

f) Find the order of the pole at $z = \frac{\pi}{4}$ of the function $f(z) = \frac{1}{cosz - sinz}$

2. a) Find the Mobius transformation which maps the circle $|w| = 1$ into the circle $|z-1| = 1$ and maps $w = 0, w = 1$ into $z = \frac{1}{2}, z = 0$, respectively.

b) Prove that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and hence find its harmonic conjugate. 4 + 4

3. a) Prove that a function which has no singularity in the finite part of the plane or at infinity is constant.

b) Evaluate: $\int_C \frac{e^{az}}{(z-\pi i)} dz$ where C is the ellipse $|z-2| + |z+2| = 6$. 4 + 4

4. a) Prove that $e^{\frac{c}{2}(z-\frac{1}{z})} = \sum_{-\infty}^{\infty} a_n z^n$ where $a_n = \frac{1}{2\pi}\int_0^{2\pi} \cos(n\theta - c\sin\theta)\, d\theta$.

b) If a function $f(z)$ is analytic within and on a closed contour $C: |z-a| = R$ and if $|f(z)| \le M$ for every z on C then show that $|f^n(a)| \le \frac{Mn!}{R^n}$. 5 + 3

5. a) Evaluate $\int_0^{\pi} \left(\frac{1+\cos\theta}{4+5\cos\theta}\right) d\theta$.

b) If the mapping $w = f(z)$ is conformal then show that $f(z)$ is an analytic function of z. 4 + 4

6. a) Find linear transformation that maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$ respectively.

b) Using the calculus of residue evaluate $\int_0^{\infty} \frac{x\sin 2x}{x^2+3} dx$. 3 + 5

7. a) Prove that the zeros of an analytic function are isolated

b) Find the Laurent's series of the function $f(z) = \frac{\sin z}{\left(z-\frac{\pi}{4}\right)^3}$ in the and $0 < \left|z - \frac{\pi}{4}\right| < 1$. 4 + 4

[Internal Marks – 10]
