M.Sc. First Semester End Examination, 2023
(Regular & Supplementary Paper)
Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-103
[Ordinary Differential Equation and Special Functions]

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate mark.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary

Answer question no. 1 and any four from the rest

1. Answer any four questions: $2\times4=8$

a) Consider the second order homogeneous linear differential equation. $a_0(x)\dfrac{d^2y}{dx^2}+a_1(x)\dfrac{dy}{dx}+a_2(x)y=0$

When $a_0(x),a_1(x),a_2(x)$ are continuous function on a real interval $a\le x\le b$ and $a_0(x)\ne 0$ for all $x\in[a,b]$. Let f_1 and f_2 are two solutions of the differential equation and has relative optims at a point $x=c\in(a,b)$ then show that they are linearly dependent.

b) Let w be the Wronskian of two linearly independent solution of ODE $2y''+y'+t^2y=0 \quad t\in\mathbb{R}$ then prove that $w(t)=ce^{-t}$ where $w(t)$ is the Wronskian of solutions.

c) Show that the equation $\frac{d^2u}{dx^2}+u=f(x) \quad 0\le x\le\pi$ with $u(o)=0$ and $u(\pi)=0$ the green's function does not exists for any arbitrary function $f(x)$.

d) Find all the singularities of the following differential equation and then classify them: $(z-z^2)\omega''+(1-5z)\omega'-4\omega=0$

e) Show that $\int_{-1}^{1}P_n(z)dz=\begin{cases}0, n\neq 0\\ 2, n=0\end{cases}$ where the symbol is the usual meaning.

f) Prove that : $F(-n;b,b;-z)=(1+z)^n$ where $F(a;b,c;z)$ denotes the hypergeometric function.

2. a) If the vector functions $\varphi_1,\varphi_2,\ldots\ldots\varphi_n$ defined as follows:

$$\varphi_1=\begin{bmatrix}\varphi_1\\ \varphi_{11}\\ .\\ \varphi_{n1}\end{bmatrix},\ \varphi_2=\begin{bmatrix}\varphi_{12}\\ \varphi_{22}\\ .\\ \varphi_{n2}\end{bmatrix},\ldots\varphi_n=\begin{bmatrix}\varphi_{1n}\\ \varphi_{2n}\\ .\\ \varphi_{nn}\end{bmatrix}$$

be n solutions of the homogeneous linear differential equation $\frac{dx}{dt}=A(t)x(t)$ in the interval $a\le t\le b$ then n solutions are linearly

independent in $a\le t\le b$ iff Wronskian $W[\varphi_1,\varphi_2,\ldots\varphi_n]\neq 0\,\forall t$, on $a\le t\le b$.

b) Expand $f(z)$ in the form $\sum_{r=1}^{n}C_r\,P_r(z)$ where

$$f(z)=\begin{cases}0 \text{ if } -1<Z<0\\ 1 \text{ if } \ 0<Z<1\end{cases}$$

4+4

3. a) Find the general solution of the homogeneous system

$$\frac{dx}{dt}=\begin{pmatrix}1 & -2 & 0\\ 2 & 3 & 0\\ 0 & 0 & 2\end{pmatrix} \quad \text{where } X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

b) Using generating function for Legendre polynomial.

Prove that $(2n+1)zP_n(z)=(z+1)P_{n+1}(z)+nP_{n-1}(z)$

5+3

4. a) Solve the differential equation $x^2\frac{d^2u}{dx^2}+2x\frac{du}{dx}=x^2 \quad 0\le x\le 1$ with $u(o)$ is finite and $u(1)+u'(1)=0$ using green's function.

b) Deduce the integral formula for hypergeometric function. 4+4

5. a) Solve the differential equation

$(1-z^2)\frac{d^2w}{dz^2}+2\frac{Zdw}{dz}+n(n+1)w=0$ admits a polynomial equation at $z=0$ when n is an integer.

b) Prove that $\frac{d}{dz}[J_0(z)]=-J_1(z)$

5+3

6. Find the general solution of the ODE $2Zw''(z)+(1+z)w'(z)-kw=0$ (where k is a real constant) in series form. 8

7. a) All the eigen value of regular SL problem with $p(x)>0$ are real.

b) Consider the boundary value problem $\frac{d^2y}{dx^2}+\lambda y=0, 0\le x\le\pi$ subject to $y(0)=0,\ y'(\pi)=0$. Find the eigen values and eigen functions of the problem. 4+4

[Internal assessment – 10]
