Total Pages-05 **RNLKWC/U.G.-CBCS/VS/MTMH-DSE-502/19**

2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Fifth End Semester Examinations-2021)

MTMH-DSE-502

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Probability and statistics

1. Answer any TEN questions: **2x10=20**

a) Let $A_n = \left\{ x \in \mathbb{R} : a < x \leq a + \frac{1}{n} \right\}, n = 1, 2, 3, ...$ Show that $\{A_n\}_{n=1}^{\alpha}$ is contracting sequence and $\lim_{n \to \alpha} A_n = \phi$ (null set)

b) What do you mean by statistical regularity ?

c) Justify by an example that mutually independence implies pairwise independence but not the converse.

d) Find the distribution of the square of a Poisson μ - variate.

e) Two cards are drawn from a well-shuffled pack. Find the probability that at least one of them is diamond.

f) Assuming that A and B are equally strong chess players which of the following events is more probable ?

i) A beats B in exactly 3 out of 4 games

ii) A beats B in exactly 5 out of 8 games

g) 100 litres of water are supposed to be polluted with 10^6 bacteria. Find the probability that a sample of 1.c.c. of same water is free from bacteria.

h) Prove that $F(a+0, c) = F(a, c)$, $F(x, y)$ being joint distribution function of random variable X & Y.

i) A continuous distribution has probability density function $f(x) = ae^{-ax}, 0 < x < \alpha, a > 0$ Find the moment generating function.

j) Two random variables are connected by $aX + bY + c = 0$. Find $\rho(x, y)$

k) Prove that standard deviation is dependent on the unit of measurement but independent of the choice of origin of measurement.

l) Define distribution of sample and sampling distribution.

m) Prove that sample mean is consistent estimate of population mean.

n) If T_1 and T_2 be two statistics with $E(T_1) = 2\theta_1 + \theta_2$ and $E(T_2) = \theta_1 - 2\theta_2$. Find the unbiased estimators of θ_1 and θ_2

o) Give an example of a statistic which is unbiased estimate of population variance.

2. Answer any FOURquestions **4x5=20**

a) If X is uniformly distributed over $(0, \pi/2)$, find the expectation of $Sin\, x$. Also find the distribution of $Sin\, X$ and show that the mean of this distribution is the same as the above expectation. 2+2+1

b) The numbers X, Y are choosen at random from a set of natural number $\{1, 2, ...N\}, N \geq 3$, with replacement. Find the probability that $|X^2 - Y^2|$ is divisible by 3

c) If X is normal (m, σ) variate,prove that $P(a < X < b) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right)$ and $P(|X - m| > a6) = 2[1 - \phi(a)$ where $\Phi(x)$ denotes the standard normal distribution function.

d) There are two identical urns containing respectively 4 white and 3 red balls, 3 white and 7 red balls. An urn is chosen at random, and a ball is drawn from it. Find the probability that the ball is white. If the drawn ballis white, What is the probability that it is from the 1^{st} urn ?

e) A player repeatedly throws a coin and scores one point for a head and two points for a tail. If p_n denotes the probability of scoring n points, show that $2p_n = p_{n-1} + p_{n-2}$ Hence deduce an expression for p_n and find its limiting value as $n \to \alpha$

f) The random variable X is normally distributed with mean 68 and standard deviation 2.5. What should be the size of the sample whose mean shall not differ from the population mean by more than 1 with probability 0.95 ?

Given that $\frac{1}{\sqrt{2\pi}} \int_{1.96}^{\infty} \bar{e}^{t^2/2} dt = 0.025$

g) Two points are independently chosen at random in the interval (0, 1). Find the probability that the distance between them is less than a fixed number $k(0 < k < 1)$.

3. Answer any TWO questions **10x2=20**

a) i) If X_1, X_2 are independent random variables each having the density function $2xe^{-x^2} (0 < x < \alpha)$, Find the distribution of random variable $\sqrt{x_1^2 + x_2^2}$

ii)Find the sampling distribution of the mean for gamma population

b) i)Prove Schwartz's inequality. for expectation that $\left[E(XY)\right]^2 \leq E\left(X^2\right)\left(Y^2\right)$. Hence deduce that $-1 \leq P(x, y) \leq 1$

ii) Find the marginal density function from bivariate normal distribution.

[The End]