M.Sc. First Semester End Examination, 2023

(Regular & Supplementary Paper)

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-105

[CLASSICAL MECHANICS AND NONLINEAR DYNAMICS]

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate mark.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary

Answer question no. 1 and any four from the rest

1. Attempt any **four** questions: $2 \times 4 = 8$
   a) Define cyclic coordinate. Give one example of it.
   b) Prove that for low velocity Lorentz transformation approaches to Galilean.
   c) What is the Coriolis force, and how does it relate to a rotating frame?
   d) State the basic postulates of vthe special theory of relativity.
   e) Show that the following transformation is canonical
   $$Q = log\left(\frac{sinp}{q}\right),\ P = qcotp.$$
   f) State Hsmilton's principle and write its significance.
2. a) Given a mass-spring system consisting of a mass m and a linear spring of stiffness k hanging from a fixed point. Find the

equation of motion using the Hamiltonian procedure, assume that the displacement x is measured from the unstreched position of the spring.

b) If the equation of transformation do not depends explicitly on time and if the potential energy is velocity independent, H is the total energy of the system. 4+4

3. a) A Lagrangian for a particular physical system can be written as $L=\frac{m}{2}\left(x^2+2bxy+cy^2\right)-\frac{k}{2}\left(ax^2+2bxy+cy^2\right)$, where a, b and c are arbitrary constants satisfying $b^2-ac\neq 0$. Determine the Lagrange's equation. Examine particularly the case $a=0=c$.

b)) Derive Hamilton's equation of motion from Hamilton's principle. 4+4

4. Suppose a partical of mass m_0 is moving with a velocity v then show that its mass at any time is given by $m=\frac{m_0}{\sqrt{1-v^2/c^2}}$ where c is the speed of the light. 8

5. a) A body moves about a point O under no forces. The principal moments of inertia at O being $3A, 5A$ and $6A$. Initially, the angular velocity has components $\omega_1 = n, \omega_2 = 0, \omega_3 = n$ about the corresponding principal axes. Show that at any time t,

$$\omega_2 = \frac{3n}{\sqrt{5}}\tan\left(\frac{nt}{\sqrt{5}}\right)$$

and that the body ultimately rotates about the mean axis. 8

6. Prove that $J=\int_{x_0}^{x_1}F(y_1,y_2,\dots,y_n,y_1',y_2'\dots,y_n',x)dx$ will be stationary if $y_1,y_2,\dots,y_n$ are obtaining by solving the equations

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y_j'}\right)-\frac{\partial F}{\partial y_j}=0, j=1,2,\dots,n, \text{ where } y_j'=\frac{\partial y_j}{\partial x}.$$

b) Let $G=G_1\left(q_1,q_2,\dots\dots,q_n,Q_1,Q_2,\dots\dots,Q_n,t\right)$ be a gernerating function of a canonical transformation. Prove that $p_j=\frac{\partial G_1}{\partial q_j}$, $P_j=\frac{\partial G_1}{\partial Q_j}$ for all j. Hence, prove that if the canonical transformation is given, then one can determine the generating function. 4+4

7. What do you mean by qualitative study of a system? Find the nature of the critical point for the linear system $\dot{x}=(x+y), \dot{y}=(x-y+1)$ Also, find the phase paths of the system 2+6

[Internal Marks – 10]
