Mathematics

[MAJOR]

(NEP-CBCS)

(B.Sc. First Semester End Examinations-2023)

PAPER : MTMH-MJ-101

(Calculus, Geometry and Linear Algebra-I)

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Group – A

[Calculus: Marks-23]

1. **Answer any FOUR questions:** **4x2=8**

a) If $= sinkx + coskx$, prove that $y_n = k^n[1 + (-1)^n sin2kx]^{\frac{1}{2}}$

b) Evaluate $\lim_{x\to 0}\left(\frac{sinx}{x}\right)^{\frac{1}{x}}$

c) Find the radius of curvature of $y = xe^{-x}$ at the point where y is maximum.

d) Find the point of inflexion on the curve $r = \frac{a\theta^2}{\theta^2-1}$

e) Find the asymptote (if any) of the curve $y = a\log\left(\sec {x}/{a}\right)$.

f) Determine the length of one arch of the cycloid $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$

2) Answer any ONE question **1x5=5**

a) Prove that the envelope of circles whose centres lie on the rectangular hyperbola $xy = c^2$ and which passes through its centre is $(x^2 + y^2)^2 = 16c^2xy$

b) Let $P_n = D^n(x^n \log x)$ Prove that the recurrence relation $P_n = nP_{n-1} + (n-1)!$

Hence show that $P_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$

3) Answer any ONE question **1x10=10**

(a)(i) Find the equation of the cubic which has the same asymptotes as the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y - 1 = 0$ and which passes through the points (0,0),(1,0) and (0,1).

(ii) Find the value of a, b, c so that $\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

5+5

(b)(i) If $y = e^{m\sin^{-1}x}$ show that $(1-x^2)\, y_{n+2} - (2n+1)\, xy_{n+1} - (n^2+m^2)y_n = 0$. Also find y_n at x=0.

(ii) Find the perimeter of the Cardioide $r = a(1 - \cos\theta)$ and show that the arc of the upper half of the curve is bisected at $\theta = \frac{2\pi}{3}$.

(3+2)+5

Group – B

[Geometry: Marks-23]

4. Answer any FOUR questions: **4x2=8**

a) On the ellipse $r(5-2\cos\theta) = 21$, find the point with the greatest radius vector.

b) Find the angle of rotation about the origin which will transform the equation $x^2-y^2=4$ into $XY+2=0$

c) Determine the value of a so that the equation $ax^2+6xy+9y^2+3x+6y-4=0$ may represent a conic having no centre.

d) Find the value of c for which the plane $x + y + z = c$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$

e) Find the equation of the cone whose vertex is the origin and base is the curve $2x^2+3y^2=1, z=0$

f) Show that the equations $x=1+\lambda$ $y=-1+2z/\lambda$ represents a generator of the hyperboloid $x^2-2yz=1$

5) Answer any ONE question **1x5=5**

a) Reduce the equation $4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$ to its conical form and determine the type of the conic represented by it.

b) Find the locus of a luminous point if the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ casts a circular shadow on the plane $z=0$

6) Answer any ONE questions **1x10=10**

a) (i) Find the director circle of the conic $\frac{1}{r}=1+e\cos\theta$

(ii) If the normal at P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the principal planes in G_1, G_2, G_3 and if $PG_1^2+PG_2^2+PG_3^2=k^2$ then show that the locus of P is the curve of intersection of given ellipsoid and the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=\frac{k^2}{a^4+b^4+c^4}$ 5+5

b) (i) Find the equation of the right circular cylinder whose axis is $\frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$ and radius is 2.

(ii) Find the equation of the lines of intersection of the plane $x-5y+3z=0$ with the cone $7x^2+5y^2-3z^2=0$.

Group – C

[Linear Algebra –I: Marks-14]

7. Answer any TWO questions: **2x2=4**

a) Let A be a matrix such that $(I+A)$ is non-singular. Show that A is skew symmetric if $(I-A)(I+A)^{-1}$ is an orthogonal matrix.

b) Find the condition on $a,b\in R$ so that the set $\{(a,b,1),(b,1,a),(1,a,b)\}$ is linearly dependent in R^3

c) Show that the eigen value of the idompotent matrix is either 1or 0.

8. Answer any TWO questions: **5x2=10**

a) Define rank of the matrix. Determine the conditions for the system of equations has only one solution, many solutions, no solution: $ax+y+z=6, x+ay+2z=1,\ x+y+az=1$.

b) State cayley-Hamilton theorem and apply this to show that for the matrix $A=\begin{bmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{bmatrix}$, A^{-1} is a polyunomial in A and also find A^{-1}

c) Prove that eigen values of orthogonal matrix is unit modulus.