Mathematics [Minor]

(NEP-CBCS)

(B.Sc. First Semester End Examinations-2023)

PAPER: MTM-MI-01

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Group – A

[Calculus: Marks-23]

1) Answer any FOUR questions: **4x2=8**

a) Evaluate $\lim_{x \to 0} \frac{\log(1-x^2)}{\log\cos x}$

b) Evaluate $\int_0^{\pi/2} \sin^{3/2} x \cos^3 x\, dx$

c) If $I_n = \int e^{-x} x^n dx$, prove that $I_n = -e^{-x}x^n + nI_{n-1}$

d) Find all the asymptotes of $x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$.

e) Use Leibnitz's rule to find the nth derivative of $x^2e^x \cos x$

f) Find the point of inflexion for the curve $y = x^3$ at $x = 0$.

2) Answer any ONE question: **1x5=5**

a) Find the reduction formula for $I_{m,n} = \int_0^{\pi/2} \cos^m x \sin nx\,dx, m, n$ being positive integers. And hence deduce that

$$I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \cdots + \frac{2^m}{m}\right].$$

b) (i) Find the total length of the astroid $x = a\cos^3\theta, y = a\sin^3\theta$

(ii) Evaluate $\lim\limits_{x\to 2} \dfrac{x^3 - 2x^2 + 2x - 4}{x^2 - 5x + 6}$

3) Answer any ONE question: **1x10=10**

(a) (i) If $y = a\cos(\log x) + b\sin(\log x)$

Prove that $x^2 y_2 + x y_1 + y = 0$

& $x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2+1) y_n = 0$

(ii) Find the radius of curvature of $9x^2 + 4y^2 = 36x$ at the point (2,3). 6+4

(b)(i) Show that the arc of the upper half of the Cardioide $r = a(1-\cos\theta)$ is bisect at $\vartheta = \dfrac{2\pi}{3}$. Show that the perimeter of the curve is 8a.

(ii) Show that $y = x^4$ is concave upward at the origin and $y = e^x$ everywhere concave upward. (3+3)+4

Group – B

[Geometry: Marks-23]

4) Answer any FOUR questions: **4x2=8**

a) Determine the nature of the conic $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$.

b) Find the centre and the radius of the sphere $3x^2 + 3y^2 + 3z^2 + 2x - 4y - 2z - 1 = 0$

c) Find the equation of the cone whose vertex is origin and the base is $y^2 + z^2 = b^2, x = a$

d) Find the translation which transforms the equation $x^2 + y^2 - 2x + 14y + 20 = 0$ into $x'^2 + y'^2 = 30$.

e) Find the equation of the quadratic cylinder with generators parallel to z – axis and passing through the curve $ax^2 + by^2 + cz^2 = 1, lx + my + nz = p$

f) Find the point on the conic $\dfrac{15}{r} = 1 - 4\cos\theta$ whose radius vector is 5.

5) Answer any ONE question **1x5=5**

a) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ and $2x + 3y + 4z = 8$ is a great circle.

b) If g is a variable tangent of the conic $l/r = 1 - e\cos\theta$, show that the locus of the perpendicular from the pole on g is the circle $r^2(e^2-1)+2elr\cos\theta+l^2=0$

6) Answer any ONE question **1x10=10**

a) (i) Reduce the equation $x^2-5xy+y^2+8x-20y+15=0$ to its standard canonical form and show that it represents a hyperbola.

(ii) Find the nature of the conic $\frac{8}{r}=4-5\cos\theta$ 7+3

b) (i) Find the equation of the cone whose vertex is the point (1,2,3) and guiding curve is the circle $x^2+y^2+z^2=9,\ \ x+y+z=1$

(ii) Find the vaues of c for which the plane $x+y+z=c$ touches the sphere $x^2+y^2+z^2-2x-2y-2z-6=0$. 6+4

Group – C

[Linear Algebra – I: Marks 14]

7) Answer any TWO questions: **2x2=4**

a) Prove that the set of vectors {(1,2,2), (2,1,2), (2,2,1)} is linearly independent in $\mathbb{R}^3$

b) Find the rank of the matrix $\begin{pmatrix}1 & 0 & 3\\ 4 & -1 & 5\\ 2 & 0 & 6\end{pmatrix}$.

c) Find the eigen value and eigen vector for the matrix $A=\begin{pmatrix}1 & 3\\ 4 & 5\end{pmatrix}$

8) Answer any TWO questions: **2x5=10**

a) State and prove caley Hamilton theorem.

b) Solve the system of equation

$$x_1+3x_2+x_3=0$$
$$2x_1-x_2+x_3=0$$

c) Find the geometric and algebraic multiplicities of each eigen value of the matrix $\begin{pmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{pmatrix}$