Mathematics [SEC]

(NEP-CBCS)

(B.Sc. First Semester End Examinations-2023)

PAPER: MTM-SEC01

Full Marks: 40 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

1) Answer any FIVE questions: **5x2=10**

a) Prove the following by contradiction: If 10 balls are choosen from a collection of red, green and yellow balls, then there are at least 5 red balls, 4 green balls and 3 yellow balls.

b) Define tautology.

c) Prove $\left[(A\cup(B\cap C))\cap(A'\cup(B\cap C))\right]\cap(B'\cup C')=\phi$

d) Let $f:A\to B$ be a function from a Boolean algebra A to a Boolean algebra B. Prove that f is a Boolean algebra homomorphism if $f(a\vee b)=f(a)+f(b)$, $\overline{f(a')}=\overline{f(a)}$

e) Determine the cardinality of the set $A=\{x\in Q\,|\,x^3-x^2-2x+2=0\}$

f) For every set E, show that $(\wp(E),\cap,\cup,\subseteq)$ is a bounded lattice.

g) Draw the Hasse diagram for the set of positive integral divisors of 210 when ordered by divisibility.

2) Answer any FOUR questions: **4x5=20**

a) (i) Construct the truth table of the following

$(P \vee Q) \rightarrow ((P \vee R) \rightarrow (R \vee Q))$

(ii) Prove that $(P \rightarrow Q) \leftrightarrow \neg P \vee Q$ is a tautology.

b) Show that the following distributive laws hold in a distributive lattice.

(i) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(ii) $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$

c) (i) Find the number of integers between 1 and 250 that are not divisible by 2 or 7 but divisible by 5.

(ii) State principle of Inclusion-Exclusion 4+1

d) (i) Let L_1 ands L_2 be lattices with partial orders $\leq_1$ *and* $\leq_2$ respectively. Define $(a_1, b_1) \leq (a_2, b_2)$ if $a_1 \leq a_2$ *and* $b_1 \leq b_2$ show that $(L_1 \times L_2, \leq)$ is a lattice.

(ii) Let $(L, \leq)$ be a lattice. Then for any $a, b \in L$ prove that $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b$. 3+2

e) Let $Q(x,y,z)$ be the statement "$x + y = z$". What are the truth values of the statements $\forall x \forall y \exists z Q(x, y, z)$ and $\exists z \forall x \forall y Q(x, y, z)$, where the domain of all variables consist of all real numbers?

f) (i) 11 numbers are choosen from $\{1, 2 \cdots 20\}$ Show that one of them will be a multiple of other.

(ii) Find the number of subsets of $\{1, 2 \cdots 10\}$ by having more than four elements.

(iii) Define Lattice. 2+2+1

3) Answer any ONE question **1x10=10**

(a) (i) Prove by induction

$$\frac{1^2}{1.3} + \frac{2^2}{3.5} + \cdots \text{upto n terms} = \frac{n(n+1)}{2(2n+1)}$$

(ii) Find the numbers of elements in the union of four sets A, B, C, D having 150, 180, 210 and 240 elements respectively, given that each pair of sets has 15 elements in common, each triple of sets has 3 elements in common and $A \cap B \cap C \cap D = \phi$

(iii) A relation R on the set of non-zero complex numbers is defined by $Z_1 R Z_2$ .if and only if $\frac{z_1 - z_2}{z_1 - z_2}$ is real, show that R is an equivalence relation. 4+3+3

(b) For the logic circuit given below, do the following:

(i) Determine the Boolean expression corresponding to the circuit

(ii) Simplify the Boolean expression as much as possible, using the laws of Boolean Algebra

(iii) Draw the logic circuit corresponding to your simplified expression. 10