

2023
B.Sc. (Honours)
B.Sc. Fifth Semester End Examination - 2023
PHYSICS
PAPER - CC11T

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own
words as far as practicable.
Illustrate the answers wherever necessary.

Group - A

1. Answer any five questions : 5×2=10

(a) Prove that for any three operator $\hat{A}, \hat{B}, \hat{C}$

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$$

- (b) Find the energy of second excited state of a particle of mass m trapped in one-dimensional box of length ℓ .
- (c) What do you mean by "stationary states"?
- (d) Consider the ground state wavefunction of hydrogen atom.

(Turn Over)

(2)

$$\psi_0^{Z=3} = Ae^{-r/a_0}$$

- (e) Can Lithium ($Z=3$) give rise to Normal Zeeman effect? Explain.
- (f) Prove that the eigen values of a Hermitian operator are real.
- (g) In Stern Gerlach experiment why is it necessary to use a beam of neutral atoms and not of ions?
- (h) Find out the magnetic moment of an atom in the state ${}^2D_{3/2}$.

Group- B

Answer any four questions.

4×5=20

2. Show that the uncertainty product of position and momentum for n^{th} stationary state of infinite square well is given by

$$\Delta x \Delta p = \frac{\hbar}{2} \sqrt{\frac{n^2 \pi^2}{3} - 2}$$

3. A particle in the infinite square well has the initial wavefunction.

$$\psi(x, t=0) = A \sin^3\left(\frac{\pi x}{a}\right) \text{ for } 0 \leq x \leq a$$

1. Determine the normalization constant.
2. Calculate the expectation value of position.

(3)

4. A particle in an infinite potential box with walls at $x=0$ and $x=a$ has following wavefunction at some initial time

$$\psi(x, 0) = \frac{1}{\sqrt{5a}} \sin\left(\frac{\pi x}{a}\right) + \frac{2}{\sqrt{5a}} \sin\left(\frac{3\pi x}{a}\right)$$

- (i) Find the possible results of measurement of the system's energy and corresponding probabilities.
 - (ii) Find the expectation value of energy. 2+3
5. State and prove the Ehrenfest Theorem. 1+4
6. What is Larmor Precession? Draw the relevant diagram and derive the expression for Larmor frequency. 1+1+3
7. What is spin orbit coupling? Explain the fine structure splitting in the energy levels due to this. 1+4

Group - C

Answer any one question.

1×10=10

8. Consider a quantum oscillator of mass m which is kept in a potential

$$V(x) = \frac{1}{2} m \omega^2 x^2$$

where ω is classical angular frequency. Also consider two operators given by

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m \omega}} (\mp i \hat{p} + m \omega \hat{x})$$

(4)

1. Show that $[\hat{a}_-, \hat{a}_+] = 1$
2. Prove that if Ψ satisfies the Schrodinger equation with energy E , then $\hat{a}_\pm \Psi$ also satisfies the Schrodinger equation with energy $E \pm \hbar\omega$.
3. Show that the normalised ground state of this quantum harmonic oscillator is

$$\psi_{\text{ground}} = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left\{-\frac{m\omega}{2\hbar}x^2\right\}$$

4. Show that the expectation value of potential energy in n^{th} state of the harmonic oscillator is given by

$$\langle V \rangle = \frac{1}{2} \hbar\omega \left(n + \frac{1}{2}\right)$$

9. The normalized wave function for the ground state of hydrogen like atom is

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$

1. Calculate the most probable distance.
2. Sketch the radial probability distribution function $P_{100}(r)$.
3. Calculate the average distance of the electron from the nucleus.

4+(3+1+2)