Mathematics

[Honours]

(B.Sc. Fifth SemesterEnd Examination-2023)

PAPER-MTMH C502

[Partial Differential Equation & Metric Space - II]

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Use separate answer script for each group]

Group-A

[Partial Differential Equation: Marks – 42]

1. Answer any SIX questions $6 \times 2 = 12$

a) What do you mean by singular solution of a PDE.

b) Construct a PDE from the equation $u = ae^{-b^2t}\cos bx$, where a and b are arbitrary parameters.

c) Discuss the geometrical interpretation of the solution of Lagrange's equation $Pp + Qq = R$, where p and q have their usual meanings.

d) Obtain the region in which the following PDE is hyperbolic:

$$yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y.$$

e) Find the characteristic curves of the equation $\frac{\partial^2 z}{\partial x^2} = x^2\left(\frac{\partial^2 z}{\partial y^2}\right)$

f) Let $u = u(x,t)$ be a solution to the IVP $u_{tt} = u_{xx}$ for $-\infty < x < \infty,\ t > 0$

with $u(x,0) = \sin x,\ \ u_t(x,0) = \cos x$, then obtain the value of $u\left(\frac{\pi}{2}, \frac{\pi}{6}\right)$.

g) What is the heat equation and what does it describe?

h) Find the P.I. of $(D^2 + DD')z = Cos\,h(x+y)$.

i) Solve the PDE $(2D^2 - D'^2 + D)u = 0$ where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$.

2. Answer any TWO questions **$2 \times 5 = 10$**

a) Find complete and singular integrals of the $PDE: 2xz - px^2 - 2qxy + pq = 0$

b) Find the integral surface of the linear PDE

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

Which contains the straight line $x + y = 0, z = 1$.

c) Solve $(3D^2 - 2D'^2 + D - 1)z = 4e^{x+y}Cos(x+y)$

3. Answer any TWO questions **$2 \times 5 = 10$**

a) Classify and reduce the PDE $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ to a canonical form and hence solve it. 10

b) (i) Find the complete integral of $2(z + xp + yq) = yp^2$

(ii) Find the solution of the heat equation

$$\frac{\partial u}{\partial t} - 3\frac{\partial^2 u}{\partial x^2} = 0, \quad 0 \le x \le 2,\ t \ge 0$$ 10

subject to the boundary conditions $u(0,t) = u(3t) = 0$, and initial condition $u(x,0) = x,\ 0 \le x \le 2,\ t \ge 0$ where $u(x,t) < \infty$ as $t \to \infty$

c) (i) Show that the solution of the wave equation

$$u_{tt} = c^2 u_{xx},\ 0 < x < L, \qquad t > 0$$

satisfying the ICs:

$$u(x,0) = f(x), \qquad 0 \le x \le L$$

$$u_t(x,0) = g(x), \qquad 0 \le x \le L$$

and the BCs:

$$u(0,t) = u(L,t) = 0$$

where $u(x,t)$ is twice continuously differentiable function with respect to x and t, is unique.

(ii) Find the solution of the nonlinear PDE $p^2z^2 + q^2 = 1$. 5+5

Group-B
[Metric Space – II Marks -18]

1. **Answer any FOUR questions** **$4 \times 2 = 8$**

a) Let *A*, *B* be subsets of a metric space (*X*, *d*) with *B* a compact subset. Prove that $d(A, B)=0$ iff $\bar{A} \cap B \neq \varphi$

b) Give an example of a subsets of real numbers **R** with usual metric which is both connected and compact and which is neither connected nor compact.

c) Prove that a connected subset of the set of real number **R** with usual metric is always compact.

d) Prove that every contraction mapping is continuous.

e) What is open cover and give an example of open cover of **R**.

f) Prove that singleton subset of any matric space is alwase connected.

2. **Answer any TWO questions** **$2 \times 5 = 10$**

a) A mapping f from a matric space (X, *d*) to a metric space (Y, d') is continuous on *X* if and only if $\overset{-1}{f}(G)$ in an open set in *X* whenever *G* is an open set in *Y*.

b) Prove that in a metric space (X, *d*) continuous image of a connected set in connected.

c) Prove that composition of two uniformly continuous functions is also uniformly continuous.