Mathematics

[HONOURS]

(CBCS)

(B.Sc. Fifth Semester End Examinations-2023)

[Regular & Supplementary Paper]

MTMH-DSE-501

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Linear Programming Problem

1. **Answer any TEN questions** **10x2=20**

a) Find a basic for E^3 that contains the vectors (1,2,2) and (2,0,1).

b) Find the dual of the LPP

Min $Z = 3x_1 - 2x_1$ subject to $2x_1 + x_2 \le 1$

$-x_1 + 3x_2 \ge 4,\ x_1, x_2 \ge 0$

c) Prove that the solution of the transportation problem has always a solution.

d) **Prove that, the following pay-off matrix has no soddle point:**

		B I	B II	B III
A	I	4	6	2
A	II	1	4	6
A	III	3	2	6

e) Explain what is meant by degeneracy in L.P.P

f) Define pure strategy and mixed strategy.

g) Define convex set and extreme points.

h) Rewrite the following inequations in the form of equations

$$x_1 - 2x_2 + 5x_3 \geq 6$$
$$2x_1 + x_2 - 7x_3 \leq 5$$

, $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

i) Use dominance to solve the pay-off

	B		
A	6	8	6
	4	12	2

j) A hyperplane is given by the equation $3x_1 + 2x_2 + 4x_3 + 6x_4 = 7$. Find in which half space are the points (-6,1,7,2) and (1,2,-4,1) ?

k) **Write down the mathematical formulation of assignment problem.**

l) Find the extreme points, if any of the set $S = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 1, x_1 \geq 0, x_2 \geq 0\}$

m) State fundamental theorem of LPP.

n) Prove that intersection of two convex sets is also a convex set.

o) What is the convex hull of the set

$$X = \left\{(x, y) / \frac{x^2}{3} + \frac{y^2}{2} = 1\right\}?$$

2. Answer any FOUR questions **4x5=20**

a) $x_1 = 2, x_2 = 3, x_3 = 1$ is a feasible solution of the LPP

$$Max\ z = x_1 + 2x_2 + 4x_3$$
$$Subject\ to\ 2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$
$$x_1, x_2, x_3 \geq 0$$

Find a basic feasible solution.

b) Solve the LPP $Min\ Z = 4x_1 + 8x_2 + 3x_3$

$$Subject\ to\ x_1 + x_2 \geq 2$$
$$2x_1 + x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

c) Find the optimal assignment of the assignment problem

	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

d) If $\vec{x}$ be any feasible solution to the optimal problem and $\vec{v}$ be any feasible solution to the dual problem, then show that $\vec{c}\vec{x} \le \vec{b}'\vec{v}$, notations have their usual meaning.

e) Show that the set of all convex combinations of a finite number of points is a convex set.

3. Answer any TWO questions **2x10=20**

a) Solve the Travelling salesman problem.

	1	2	3	4	5
1	∞	6	12	6	4
2	6	∞	10	5	4
3	8	7	∞	11	3
4	5	4	11	$-\infty$	5
5	5	2	7	8	∞

b) Solve the LPP by Big M method

Max $z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \ge 2, 3x_1 - 4x_2 \le 3$

$x_2 + 3x_3 \le 5,\ x_1, x_2, x_3 \ge 0$

c) Solve the game problem as an LPP

B

A	1	-1	-1
	-1	-1	3
	-1	2	-1

d) Use two phase simplex method to solve.

Min $Z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \le 3$

$2x_1 + x_3 \ge 4$

$x_1, x_2, x_3 \ge 0$

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