Mathematics

[HONOURS]

(CBCS)

(B.Sc. Fifth Semester End Examinations-2023)

MTMH-DSE-502

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Probability and Statistics

1. **Answer any TEN questions:** **10x2=20**

a) For any n events $A_1, A_2, \ldots\ldots\ldots, A_n$, Show that $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$

b) Find the moment generating function of the uniform distribution over the interval (-a, a).

c) Distinguish between sampling distribution and the distribution of sample.

d) Two persons toss an unbiased coin n times each. Show that probability of their scoring the same number of heads is $\binom{2n}{n}2^{-2n}$

e) Prove that two dimensional distribution function is monotonic non-decreasing in x and y.

f) There are 500 misprints in a book of 500 pages. What is the probability that a given page will contain at most 3 pages?

g) If $x = 4y + 5$ and $y = kx + 4$ are the regression lines of x on y and of y on x, then show that $0 < K \leq 0.25$

h) If A_n denotes the half open interval $a<x \leq a +1/n$ $(n=1,2,3,\ldots\ldots)$ show that $\{A_n\}$ is contracting sequence and lim A_n is the empty set.

i) An urn contain 1 white and 99 black balls. If 1000 drawings are made with replacements, what is the probability of 10 white balls?

j) A random variable X has the p. d. f

$$f(x) = \begin{cases} ax(2-x) & 0 \leq x \leq 2 \\ 0 & otherewise \end{cases}$$

Find Var (2-3x)

k) Find the first and the third quartiles of uniform (1,6) distribution.

l) For a symmetric Binomial variable X, $E(X^2) = 18$ find the number of trials n.

m) Give a suitable statistic that is unbiased estimate of population variance.

n) Define best critical region and power of test.

o) What is the advantage of interval estimation over point estimation?

2. Answer any FOUR questions: 4x5=20

a) Derive the regression line of y on x

b) A computer while calculating correlation coefficient between two variables x and y from 25 pairs of observation, obtain the following results: n=25, $\sum x$ =125, $\sum y$ =100, $\sum x^2$ =650, $\sum y^2$ =460, $\sum xy$ =508. It was however later discovered at the time of checking that it had copied two pairs as

x	y
6	14
8	6

while the correct values were

x	y
8	12
6	8

Find the correct values of correlation co-efficient.

c) Prove that variance of the t-distribution with n degrees of freedom exists if n>2 and that value is n/(n-2).

d) Determine the probability mass function $f(x) = p(X = x)$ from the relation $f(x) = \frac{\lambda}{x} f(x-1)$ for $x = 1, 2, 3, \ldots$

where $f(x) > 0$ *for* $x = 0, 1, 2, 3 \cdots$

e) On the basis of random sample, find the maximum likelihood estimate of the mean and variance of normal (m, σ) population. What can you say about these estimates?

f) (f) Mr. A and Mrs B plan to meet at a station between 12.00 noon and 1 PM, each agreeing not to wait more than 10 minutes for the other. They arrive at the station independently between 12.00 noon and 1.00 PM. Find the probability that they will meet.

3. Answer any TWO questions: 2x10=20

a) (i) Find the mean and variance of Pascal distribution

$$f_i = \frac{1}{1+\mu}\left(\frac{\mu}{1+\mu}\right)^i, \ \mu > 0, i = 0,1,2,\ldots\ldots\ldots$$

(ii) Find the sampling distribution of the sample mean for the gamma population. 5+5

b) (i) The heights of 10 male students of a normal population are found to be 70, 67,62, 67, 61, 68, 70,64,65, 66 inches.

Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level. Assuming for 9 degrees of freedom $P(t>1.83)=0.05$

(ii) If $X_1, X_2, \ldots\ldots, X_n$ are n mutually independent standard normal variates then prove that $X_1^2+X_2^2+\ldots\ldots\ldots\ldots+X_n^2$ is χ^2 distributed with n degrees of freedom. 5+5

c) (i) In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of perfectness (Ho : probability of success = ½) is rejected if and only if more than 4 heads are obtained. Find the probability of type – I error. Find the probability of type II error when corresponding probability of head is 0.2.

(ii) Let X_1, X_2, X_3 be a random sample of size 3 (independent) from a normal population with mean μ and variance σ^2. Let T_1, T_2, T_3 are the estimators used to estimate μ where $T_1=X_1-X_2+X_3$, $T_2= 2X_1+2X_2-3X_3$ and $T_3=(kX_1+X_2+X_3)/3$.

Are T_1 and T_2 unbiased estimators? For what value of k, T_3 is unbiased? Which is the best estimator? 3+(2+2+3)