M.Sc. Third Semester End Examination, 2023

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-301

Full Marks: 50 *Time: 02 Hrs*

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Partial differential Equation and Generalized Functions]

Answer question no. 1 and any four from the rest

1. Answer anyfour questions: 4x2=8

   a) What are the main differences between an ODE and PDE?

   b) Discuss the nature of PDE $\left(x^2-1\right)u_{xx}+2yu_{xy}-u_{yy}=0$.

   c) Show by using weak maximum and weak minimum principles that the Dirichlet problem for the Poisson's equation has unique solution.

   d) Define characteristic curve and base characteristics of a first order quasi-linear PDE.

   e) Describe 'spherical mean' of a function.

f) Find the solution of $p^2 + q^2 = x + y$

2. Classify and reduce the PDE 8

$$y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} - \frac{y^2}{x} u_x - \frac{x^2}{y} u_y = 0$$

to a canonical form and hence solve it.

3. a) Solve the PDE

$$(D^2 - 4DD' + 4D'^2)u = e^{2x+y}$$

b) Show that the following PDEs

$$xq - yq = x \text{ and } x^2 p + q = xz$$

are compatible and hence find solutions. 4+4=8

4. a)Using the method of separation of variables find a formal solution of the problem

$$\begin{aligned} &u_{tt} = u_{xx},\ 0 < x < \pi,\ t > 0, \\ &u(0,t) = u(\pi,t) = 0, t \geq 0, \\ &u(x,0) = \sin^3 x, 0 \leq x \leq \pi, \\ &u_t(x,0) = \sin 2x, 0 \leq x \leq \pi, \end{aligned}$$

b) Prove that every nonnegative harmonic function in the disk of a radius a satisfies the following: 4+4

$$\frac{a-r}{a+r} u(0,0) \leq u(r,\theta) \leq \frac{a+r}{a-r} u(0,0)$$

5. Obtain the solution of the interior Neumann problem for a circle describe by 8

$$\text{PDE: } \nabla^2 U = 0,\ 0 \leq r < a;\ 0 \leq \theta < 2\pi$$

$$\text{BC : } \frac{\partial u}{\partial n} = \frac{\partial u(a,\theta)}{\partial n} = g(\theta),\ r = a$$

6. a) A string of length L is released from rest in the position $y = f(x)$ Show that the total energy of the string is

$$\frac{\pi^2 T}{4L} \sum_{n=1}^{\infty} n^2 k_n^2$$

Where $k_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$ and T is the tension of the string.

b) Prove that the Neumann problem for a bounded region has a solution, then it is either unique or it differs one another by a constant 5+3

7. a) Prove that $\delta(at) = \frac{1}{t}\delta(t),\ a > 0$

(Symbols have their usual meaning).

b) Solve the following problem:

$$\begin{aligned} &u_t = u_{xx} - u, 0 < x < 1,\ t > 0 \\ &u(0,t) = u_x(1,t) = 0, t \geq 0 \\ &u(x,0) = x(2-x), 0 \leq x \leq 1 \end{aligned}$$

3+5=8

Internal Assessment - 10