M.Sc. Third Semester End Examination, 2023

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-302

Full Marks: 50 *Time: 02 Hrs*

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Transform and Integral Equation]

Answer question no. 1 and any four from the rest

1. **Answer any four questions:** **4x2=8**

a) Find $f(o)$and $f'(x)$ where $F(p)=\frac{2p}{p^2-2p+s}$. Here is Laplace transform of $f(t)$

b) If F(k) be the Fourien transform of $f(x)$ then prove that $F(k+a)$ is the Fourien transform of $e^{iax}f(x)$ where a is real.

c) Evaluate the Convolution of two functions $\frac{1}{\sqrt{\Pi t}}$ and e^{at}

d) Find $L^{-1}\left[\frac{p^2-2}{(p-1)^4}\right]$

e) Show that the function $u(x)=1-x$ is a solution of the integral equation $u(x)=x+\int_0^1 (x-t)^2 u^2(t)dt$

f) If the function $f(t)$ has period $T>0$ then calculate $L\{f(t)\}$ in the simplest form.

2. a) If in any finite interval a function $f(x)$ is continuous and its derivative is piece-wise continuous, the integrals and $\int_{-\infty}^{\infty} f(x)dx$, $\int_{-\infty}^{\infty} f'(x)dx$ are absolutely convergent and $f(x)\to 0$ as $|x|\to\infty$ then prove that $F[f'(x)]=-ik\,F(k)$ where F(k) is the Fourier transform of f(x).

b) Find the resolvent kernel and using this solve the integral equation $\phi(x)=x+\int_0^1 (t+x)\phi(t)dt$ 4+4=8

3. a) If $f(t)$ is continuous and is of exponential order $o(e^{at})$ as $t\to\infty$ and $f'(t)$ is piecewise continuous in any interval of t then $\lim_{p\to 0} p\,F(p)=\lim_{t\to\infty} f(t)=f(\infty)$. $F(p)$ is the Laplace transform of f(t)

b) If a and b are real constants, solve the following integral equation: $ax+bx^2=\int_0^x \frac{y(t)}{(x-1)^{\frac{1}{2}}}dt$ 4+4=8

4. a)By use of Laplace Transform find the solution of the equation

$$t\frac{d^2y}{dt^2}+2\frac{dy}{dt}+ty=\sin t$$

b) Use Perseval's relation to show that $\int_{\partial}^{\infty}\frac{\sin ax\sin bx}{x^2}dx=\frac{\pi a}{2}$ where $o<a<b$ 4+4=8

5. Find the solution of the following problem of free vibration of a stretched string of an infinite length

$$\frac{\partial^2 u}{\partial x^2}-\frac{1}{c2}\frac{\partial^2 u}{\partial t^2}=0,\ -\infty<x<\infty$$

subject to $u(x,0)=f(x)$, $\frac{\partial}{\partial t}u(x,0)=g(x)$, u and $\frac{\partial u}{\partial x}$ both vanish as $|x|\to\infty$. 8

6. a) Use residue theorem find the inverse of the function

$$\frac{p}{(p-2)(p^2+4)}$$

b) Solve $\phi(x)=x+\int_0^x (t-x)\phi(t)dt$ by iterative kernal method.

7. a) From an integral equation corresponding to the following differential equation $\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+e^{-x}y(x)=x^3-5x$, subject to conditions, $y(0)=-3$ and $y'(0)=4$:

b) If $L\{f(t)\}=F(p)$ which exists Real $(p)>\gamma$ and $H(t)$ is unit step function, then prove that for any $\alpha, L\{H(t-\alpha)f(t-\alpha)\}=e^{-p\alpha}F(p)$ which exists for Real $(p)>\gamma$. 5+3

Internal Assessment - 10
