Total Pages-05 RNLKWC/P.G./IIIS/MTM-305A&B/23

M.Sc. Third Semester End Examination, 2023

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-305 A & B

Full Marks: 100 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

USE SEPARATE ANSWER SCRIPT FOR TWO UNITS

Unit – I

MTM-305A

[Dynamical Meteology - I]

Full Marks - 50

Attempt any **five** questions $8 \times 5 = 40$

1. Derive Gibb's general thermodynamical relation for sea water.

Prove that $C_v = C_p + T \left(\frac{\partial \tau}{\partial T}\right)^2 \Big/ \left(\frac{\partial \tau}{\partial p}\right)$ (symbols have their usual meaning)

2. Find the condition of stability of equilibrium of stratified fluid, and hence explain the significance of the Brunt- Väisalä frequency.

3. Express the principle of conservation of mass in the form of following pair of equations

$$\frac{Dp}{Dt}+\rho\, div\,\vec{q}=u$$

$$\rho\frac{Ds}{Dt}=-div\vec{I}s$$

In usual notation, assuming sea-water to be a two component mixture of salt and pure water.

4. Derived the linearised equations of small amplitude oceanian wave motion on a rotating earth.

5. State the Boussinesq's approximation. Using this approximation, modify the energy equation.

6. Obtain the equation of motion of sea-water in the form

$$\frac{d\vec{q}}{dt}=\bar{F}+2\vec{q}\times\vec{\Omega}-\frac{1}{\rho}\nabla p+\frac{1}{p}(\lambda+\mu)\nabla\theta+\gamma\nabla^2\vec{q}$$

Where $\theta=\nabla.\vec{q}$ and the other symbols have their usual meaning.

7. Derive the Ertel's Formula for potential vorticity. Prove that the potential vorticity is an approximate adiabatic invariant for sea water.

Internal Assessment - 10

Unit – II

MTM-305B

[Advanced Optimization and Operation Research]

Full Marks – 50

Answer Q. 1 and any four from rest of questions.

1. **Answer any four questions** **4 × 2 = 8**

(a) State the necessary and sufficient conditions for a stationary point of a multivariable optimization problem with equality constraints.

(b) What are the initial criteria to apply dual simplex method and what is the achievement of this method?

(c) What is the necessity of study the "Post Optimality Analysis"?

(d) What is golden ratio? What is the value of golden number?

(e) Write down the iterative scheme of Steepest Descent method.

(f) Discuss the different types of achievement in goal programming problem.

2. Solve the following goal programming problem:

Maximize $z=P_1d_1^-+P_2\left(2d_2^-+3d_3^-\right)$

Subject to

$20x_1 + 10x_2 \le 60$
$10x_1 + 10x_2 \le 40$
$40x_1 + 80x_1 + d_1^- - d_1^+ = 600$
$x_1 + d_2^- - d_2^+ = 2$
$x_2 + d_3^- - d_3^+ = 2$
$x_1, x_2, d_1^-, d_1^+ \ge 0, i = 1,2,3$ 8

3. What are the limitations of Fibonacci method? Using Fibonacci method

minimize $f(x) = \begin{cases} (x^2 - 6x + 13)/4 \ , x \le 4 \\ x - 2, \quad x > 4 \end{cases}$ in the interval [2,5]

taking n=5 2+6

4. Using decomposition principle, write down the following LPP to an elegant form of LPP that can be solved by simplex method.

Max z= $6x_1+7x_2+3x_3+5x_4+x_5+x_6$

$x_1+x_2+x_3+x_4+x_5+x_6 \le 50$

$x_1+x_2 \le 10$

$x_2 \le 8$

$5x_3+x_4 \le 12$

$x_5+x_6 \ge 5$

$x_5+5x_6 \le 50$

$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$ 8

5. Using Davidon -Fletcher-Powell method minimize

$f(x_1, x_2) = x_1^2 + 2x_2^2 + x_1 - 2x_2$ starting from the point $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ 8

6. Derive the conditions of the range of discrete changes of the component of cost vector (C) of the LPP

Maximize Z= CX

Subject to AX = b

And $X \ge 0$

Such that the optimal solution does not alter 8

7. Let us consider the final table of a LPP

C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
2	Y_1	3	1	0	0	-1	0	0.5	0.2	-1
4	Y_2	1	0	1	0	2	1	-1	0	0.5
1	Y_3	7	0	0	1	-1	-2	5	0.3	2
		Zj-Cj	0	0	0	2	0	2	0.1	2

When y_6, y_7 and y_8 are slack variables.

If the constraint

(a) $2x_1 + 3x_2 - x_3 + 2x_4 + 4x_5 \le 5$

(b) $2x_1 + 3x_2 - x_3 + 2x_4 + 4x_5 \le 1$

is added then find the solution of the changed LPP. 8

Internal Assessment - 10