APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING (P.G.)

M.Sc. Second Semester End Examination-2024

(Regular & Supplementary Paper)

PAPER- MTM-202

[Numerical Analysis]

Full Marks: 50 ***Time: 02Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Attempt Question No. 1 and any four from the rest:

1. Answer any **four** **4X2=8**

i) A function f(x) is defined by

$$f(x) = \begin{cases} 1 + x & \forall x \in [0,3] \\ (x-3)^2 + x + 1 & \forall x \in [3,4] \end{cases}$$

Show that f(x) satisfies the properties of a cubic spline.

ii) What is the difference between single-step and multi-step method to solve the initial value ODE problem?

iii) Show that the gaussian quadrative for (n+1) interpolating points give exact value for a degree of polynomial of degree (2n+1).

iv) If 1 is the first orthogonal polynomial with respective

to the weight function w(x)=1, then find the third orthogonal polynomial obtained by the Gram-Schmidt orthogonalization process..

v) If Tn(x) is the Chebyshev polynomial of degree n, then find the upper bound of $2^{1-n}T_n(x)$

vi) Discuss the merits and demerits of finite difference method to solve an ordinary diffential equation.

2. What do you mean by the predictor-corrector method? Explain. Describe Milne's predictor-corrector method to solve the following ordinary differential equation.

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

2+6

3. a) Discuss finel point tuition method for finding the system of equations f(x,y)=0 and g(x,y)=0 with $x(0)=x_0$ and $y(0)=y_0$. Find the condition of convergence of this method.

b) Obtain first four-orthogonal polynomial of $f_n(x)$ on [-1,1] with respect to the weight function w(x)=1 for linear independent four polynomial $\{1, x, x^2, x^3\}$ 5+3

4. Use power method to find the largest eigenvalue and corresponding eigenvector for the folklowing matrix

$$\begin{bmatrix} 5 & 3 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

8

5. a) Derive the value of $\int_a^b f(x)\,dx$ using gauss-Chebyshev quadratine formula.

b) Let f(x) be a function defined by

$$f(x) = \begin{cases} x^3 + 2x^2 + \frac{13}{3}; 0 \le x \le 1 \\ ax^3 + bx^2 + x + 4; 1 \le x \le 2 \end{cases}$$

Find the value of a and b such if f(x) be a cubic spline. 4+4

6. a) Describe LU decomposition method for the system of n equation.

b) Find the quadratic approximation to the function $y = e^{2x}$ on [0,1] by least squares method. 4+4

7. Describe the Crank-Nicoplicit method to solve the following equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 < x < 1$$

Subject to the boundary conditions $u(0,t) = f,(t)\ u(1,t) = f_2(t)$ and initial condition $u(x,0) = g(x)$ 8

[Internal Assssment-10]