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APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING (P.G.)

M.Sc. Second Semester End Examination-2024

(Regular & Supplementary Paper)

PAPER- MTM-203

Full Marks: 50 ***Time: 02Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Use separate answer script for each unit}

Unit - I

[Abstract Algebra]

Answer question No 1 and two from the rest.

Symbols have their meaning.

1. **Answer any two questions of the following:** $2 \times 2 = 4$

   (a) Verify the class equation of the symmetric group S_3.

   (b) Show that a group G of order 35 is cyclic.

   (c) Prove that there is no simple group of order 120.

   (d) Find a basis of $Q(\sqrt{3}, \sqrt{5})$ over Q.

2. (a) Let G be a non-abelian group of order $O(G) = p^3$, p is prime. Then determine $O(Z(G))$ and the number of conjugate classes of G.

(b) Suppose G be a finite commutative group of $O(G) = n$. If m is a positive divisor of n, then prove that G has a subgroup of order m.

4+4

3. (a) Let G be a group of order $O(G) = 231$. Show that 11-Sylow subgroup of G is contained in the centre of G.

(b) Describe splitting field. Show that the splitting field of the polynomial $f(x) = x^4 + 1$ over Q is $Q(\sqrt{2}, i)$ whose degree over Q is 4. 4+4

4. (a) Let $H_1, H_2, ..., H_n$ be normal subgroup in G. Then show that G is an internal direct product of $H_1, H_2, ..., H_n$ if and only if

(i) $G = H_1, H_2, ..., H_n$,

(ii) $H_i \cap H_1 H_2 ... H_{i-1}, H_{i+1} ... H_n = e$ for all $i = 1, 2, ..., n$.

(b) Let G be a group. Then show that $Inn(G)$ is normal subgroup of Aut (G). 5+3

[Internal assessment: 05]

Unit - II

[Linear Algebra]

Answer Question No. 1 and TWO from rest

1. Answer any TWO questions: $2 \times 2 = 4$

a) Let T be a linear functional on $\mathbb{R}^2$ defined by $T(2,1) = 15$ and $T(1,-2) = 10$. Find $T(x, y)$.

b) Let T be a linear operator on a finite dimentional vector space. When T is said to be diagonalizable?

c) Give an example of two self-adjoint transformations whose produce is not self-adjoint.

d) If T is normal and $T^3 = T^2$, show that T is idempotent. If normality of T is dropped, does the conclusion still true?

2. (a) Let V be a vector space of dimension n over a field F. Prove that V is isomorphic to F^n.

(b) Let T be a linear operator on $\mathbb{R}^3$ defined by

$$T\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{pmatrix}$$

Test whether T is diagonalizable or not. 4+4

3. Answer any one

(a) Show that similar matrices have the same minimal polynomial.

(b) Define adjoint operator. Prove that adjoint operator is linear.

(c) What do you mean by a normal operator? Are all self –adjoint operators normal? Justify. 3+3+2

4. (a) Let $H : v \times v \to F$ be a Hermitian form. Then Prove that a linear transformation $T : v \to v$ is H-unitary iff $H(Tx, Ty) = H(x, y) \forall x, y \in v$

(b) Let T be a normal operator on a Euclidean space E . Then prove that

i) $Tx = 0$ iff $T^* x = 0$

ii) $T - \lambda I$ is a normal operator on E 4+4

[Internal assessment: 05]