Total Pages-04 RNLKWC/M.Sc./CBCS/IIS/MTM-205/24

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING (P.G.)

M.Sc. Second Semester End Examination-2024

(Regular & Supplementary Paper)

PAPER-MTM-205

(General Theory of Continuum Mechanics)

Full Marks: 50 ***Time: 02Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any **FOUR** questions: $4 \times 2 = 8$

i) Established the relation between local rate of change and particle rate of change

ii) Explain the body forces and surfaces..

iii) If w be the complex velocity potential then find $\left|\frac{dw}{dz}\right|$

iv) What is the strain energy or stress potential?.

v) In a plane state of stress find T_{xx}, T_{yy} from the relation

$$e_{ij} = \frac{1+\sigma}{E}T_{ij} - \frac{\sigma}{E}T_{kk}\delta_{ij}$$

vi) At a point of a material the strain tensor is given by

$$E_{ij}=\begin{bmatrix}1 & 3 & -2\\ 3 & 1 & -2\\ -2 & -2 & 6\end{bmatrix}$$ Determine the principal strains.

vii) What is the principal direction of strain when the body is isotropic and the principal direction of stress is given?

viii) Is the following stress distribution possible for a body in equilibrium in the absence of body forces? $T_{11}=-Ax_1x_2$,

$$T_{12}=\frac{A}{2}\left(B^2-x_2^2\right)+Cx_3;\ T_{13}=-Cx_2;\ T_{22}=T_{33}=T_{23}=0$$

2. Answer any **FOUR** questions: **4 × 4 = 16**

(a) If Θ and θ are angle between two line elements before and after deformation then prove that $\frac{dl}{dL}\frac{\delta l}{\delta L}\cos\theta-\cos\Theta=2r_{ij}N_iM_j$, $i,j=1,2,3$ symbols have their usual meaning.

(b) The state of strain at a point in a continuum body is given by $\left(E_{ij}\right)=\begin{pmatrix}4 & -4 & 0\\ -4 & 0 & 0\\ 0 & 0 & 3\end{pmatrix}$

(c) Established the constitutive equation for isotropic elastic body and strain energy function.

(d) What is uniqueness theorem? Derive it.

(e) Established the stress and displacement relation..

(f) If the displacement field is $x_1=X_1+2X_3$, $x_2=X_2-2X_3$, $x_3=X_2-2X_1+2X_2$ then find R_i so that $R_i=e_{ijk}\frac{\partial u_k}{\partial x_j}$

(g) Give example of irrotational and rotational fluid flows..

3. Answer any **TWO** questions: **8 × 2 = 16**

i) (a) Find the Beltrami-Michell Compatibility Equations for stresses.

(b) Show that the following stress components are not solutions of a problem in elasticity, even though they satisfy equations of equilibrium with zero body forces

$T_{11}=\alpha\left[x_2^2+\sigma\left(x_1^2-x_2^2\right)\right]$ $\quad T_{22}=\alpha\left[x_1^2+\sigma\left(x_2^2-x_1^2\right)\right]$,

$T_{33}=\alpha\left[\sigma\left(x_1^2+x_2^2\right)\right], T_{12}=-2\alpha x_1x_2,\ T_{23}=T_{31}=0$ 4+4

ii) State and prove Kelvin's Minimum Energy Theorem. 8

iii) a) Prove that the displacement of any material point at any point on the strain quadric relative to that at the centre is directed along the normal to the surface of the quadric at the point.

b) Prove that all principal stresses are real. 5+3

iv) (a) $T_{ij}=\begin{bmatrix}3 & 1 & 4\\ 1 & 2 & -5\\ 4 & -5 & 0\end{bmatrix}$.; find the stress vector on a plane element through P and parallel to the plane $2x_1+x_2-x_3=1$

and the magnitude of the stress vector.

(b) Find the equation of the stream lines due to uniform line source of strength m through the point $A(-c,0)$, $B(c,0)$ and a uniform line sink strength 2m through the origin. 4+4

[Internal Assessment – 10]