APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING [P.G.]

[CBCS]

M.Sc. Fourth Semester End Examination-2024

[Regular & Supplementary Paper]

PAPER-MTM-403

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

[Use separate answer script for each unit]

Unit I
[Magneto Hydro-dynamics]
F.M. - 25

1. **Answer any two questions of the following:** **2x2=4**
   a. Describe the working principle of MHD power generator.
   b. Define magnetic Reynolds number and explain its significance.
   c. Write Alfven's theorem and its significance
   d. Explain the term 'Lorentz force' in MHD.

2. **Answer any two questions of the following:** **2x4=8**
   a. Write down the basic equations mageto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows.

b. Find the equations of motion of a conducting fluid in the context of MHD flow.

c. Prove that in a steady non-uniformly rotating star, the angular velocity must be constant over the surface traced out by the rotation of the magnetic lines of force about the magnetic field axis.

d. Define the terms Alfven's velocity and Alfven's waves Hence, derive the speed of propagation $\sqrt{c^2+V_A^2}$ for magneto-hydrodynamic wave, where symbols have their usual meaning.

3. Answer any one questions of the following: 1x8=8

a. A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate z=-L (lower) and a horizontal infinitely long non-conducting plate z=L (upper). Assume that a uniform magnetic field H_0 acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the induced magnetic field.

b. i) Find the rate of change of magnetic energy in magneto-hydrodynamic.

ii) Write down the mathematical formulation of the magneto-hydrodynamic flow past a porus plate and derive its velocity and magnetic field expressions. 3+5

[Internal Assessment – 5]

Unit II
[Stochastic Process and Regression]
F.M. - 25

Answer question no 1 and any two from the rest

1. Answer any two questions of the following: 2x2= 4

a) Discuss how a Markov Chain can be represented as a graph.

b) Define Brownian motion process.

c) When a state in a Markov chain is said to be persitant and Ergodic .

2. a) Prove that is an irreducible Markov chain all states are same type.

b) Let $\{X_n, n \geq 1\}$ be a Markov chain having state space s={1,2,3,4} and transition matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Identify the states as transient, persistent, or ergodic. 3+5

3. a) Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \geq 1$ be the digit

living the n^{th} stage of the system and X_0 be the digit entering the first stage (leaving the 0^{th} stage). At each stage, there is a constant probability q that the digits which enter will be transmitted uncharged (i.e. the digit will remain uncharged when it leaves), and probability p otherwise(i.e. the digit changes when it leaves), $p+q=1$. Find the one step transition matrix P, and n-step transition matrix P^n. Also find P^n when $n \to \infty$.

b) Define poission process with rate λ 6+2

4. a) Derive the partial differential equation of pure linear birth process and hence find $P_n(t)$ for initial population is equal to unit.

b) The correlation co-efficients r_{12}, r_{13} and r_{23} must satisfy the inequality.

$r_{12}^{\ 2} + r_{13}^{\ 2} + r_{23}^{\ 2} - 2r_{12} + r_{13} + r_{23} \leq 1$

Symbols are their usual meaning. 5+3

[Internal Assessment – 5]