Mathematics [Minor]

[NEP]

B.Sc. First Semester End Examination-2024

(Regular & Supplementary Paper)

PAPER-MTM MI101

Full Marks: 60 ***Time: 03 Hrs.***

The figures in the right hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group - A

(Calculus: Marks-23)

1. **Answer any FOUR questions:** $4 \times 2 = 8$

(a) Find the envelope of the family of straight lines $= mx + \frac{a}{m}$.

(b) Evaluate $\lim_{x \to 0} \frac{x - sinx}{x^3}$

(c) Find $\int \tan^4 x dx$.

(d) Find the point of inflexion if any of the curve $y = x^2(3 - x)$

(e) If $y = e^{m \sin^{-1} x}$, Show that

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0$$

(f) Find the radius of curvature at $\theta = \frac{\pi}{4}$ on the curve $x = a\cos^3\theta, \quad y = a\sin^3\theta$.

2. **Answer any ONE question:** $1 \times 5 = 5$

(a) Determine the asymptotes of the curve

$$(x+y)(x-2y)(x-y)^2 + 3xy(x-y) + x^2 + y^2 = 0$$

(b) Find the value of $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ by reduction formula

3. **Answer any ONE question:** $1 \times 10 = 10$

(a) (i) Find the perimeter of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

(ii) $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

(b) (i) Find the value of a, b such that $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$

(ii) Find the envelope of the lines $\frac{x}{a} + \frac{y}{b} = 1$ when the parameter a,b are connected by $a + b = c$

Group - B

(Analytical geometry: Marks-23)

1. **Answer any FOUR questions:** $4 \times 2 = 8$

(a) Find the value of λ so that the equation $x^3 + 6xy + 9y^2 + \lambda x + 12y - 5 = 0$ may represent a pair of straight lines.

(b) Discuss the nature of the conic $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$

(c) Find the polar equation of the left branch of the hyperbola $9x^2 - 16y^2 = 144$

(d) Find the equation of the cone whose vertex is the origin and base is the curve $z = 2, x^2 + y^2 = 4$.

(e) Find the equation of the sphere whose centre at (2,-3,4) and radius equal to 5 units.

(f) To what point must the origin be moved in order to remove the terms of the first degree in the equation $2x^2 - 3y^2 - 4x - 12y = 0$.

2. **Answer any ONE question:** $1 \times 5 = 5$

(a) Show that the pair of straight lines joining the origin to the other two points of intersection of the curves $ax^2 + 2hxy + by^2 + 2gx = 0$ and $a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$ will be right angles if $g'(a+b) = g(a'+b')$

(b) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9, \ x + y + z + 3 = 0$ as a great circle.

3. **Answer any ONE question:** $1 \times 10 = 10$

(a) (i) Show that the sum of the reciprocal of two perpendicular focal chords of a conic is constant.

(ii) The gradient of one of the straight lines of $ax^2 + 2hxy + by^2 = 0$ is twice that of the other. Show that $8h^2 = 9ab$. 5+5

(b) (i) Reducing the equation $x^2 - 6xy + y^2 - 4x - 4y + 12 = 0$ to its canonical form, and also determine its nature.

(ii) Find the equation of the right circular cylinder whose axis is $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$ and radius equal to 2. 7+3

Group - C

(Linear Algebra-I Marks-14)

1. Answer any TWO questions: **$2 \times 2 = 4$**

(a) Determine K so that the set

$S = \{(1,2,1),(K,3,1),(2,K,6)\}$ *in* $\mathbb{R}^3$ is linearly dependent.

(b) Show that product of the eigen values of a square matrix A is det A.

(c) Determine the rank of the matrix: $\begin{pmatrix} 2 & -2 & 0 \\ -2 & 3 & 2 \\ 0 & 2 & 4 \end{pmatrix}$

2. Answer any TWO questions: **$2 \times 5 = 10$**

(a) Investigate for what values of λ and μ the following system of equations

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + az = b^2$$

have i) no solution, ii) a unique solution.

(b) Find the eigen values and the eigen vectors of the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

(c) Verify Cayley Hamilton theorem for matrix and express A^{-1} as a polynomial in A and then compute A^{-1}

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}$$
