Mathematics [SEC]

[NEP]

B.Sc. First Semester End Examination-2024

(Regular & Supplementary Paper)

PAPER-SEC101

Full Marks: 40 ***Time: 02 Hrs.***

The figures in the right hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

1. Answer any FIVE questions: **5x2= 10**

a) Obtain a disjunctive normal form of $Pv\left(\neg P \rightarrow \left(QV\left(Q \rightarrow \neg R\right)\right)\right)$

b) Let $P = \{n^7$:n is positive integer$\}$

and $Q = \{n^{209}$:n is a positive integer$\}$

Find the cardinality of the set $P \cap Q$.

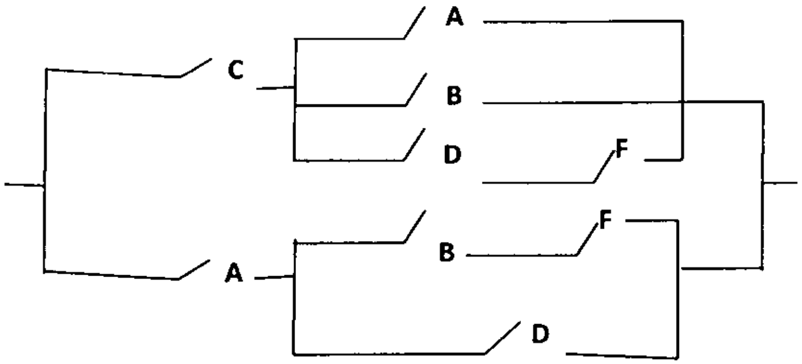
c) gf $a \le b$ and $c \le d$ for $a,b,c,d \in L$ Prove that $a \vee c \le b \vee d$ and $a \wedge c \le b \wedge d$

d) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

e) Without constructing the truth table show that the statement $(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

f) Show that the relation $\subseteq$ defined on the power set P(S) is a partial ordered relation.

g) Write the Boolean expression of the given logic circuits.



2. Answer any FOUR questions: 4x5 = 20

a) i) Define Poset.

ii) Let S be the set of all positive divisors of 72. Define a relation $\leq$ on S by "$x \leq y$ if and only if x is a divisor of y for $x, y \in S$"

Prove that $(S, \leq)$ is a poset.

iii) Draw the covering diagram of the poset. 1+2+2

b) i) Prove the identity in Boolean Algebra

$(a \cdot b)^{1} + a^{1} \cdot c + b^{1} \cdot c = a^{1} + b^{1}$

ii) Give indirect proof for the following if the sum of four real numbers less than 80, then at least one of them is less than 20.

3+2

c) i) Prove the following inequality by induction:

$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$, for $n \geq 2$.

ii) If A, B, C are three non empty subsets of U and $(A \setminus B) \cup (B \setminus A) = C$ then show that $(B \setminus C) \cup (C \setminus B) = A$ 3+2

d) (i) If R_1 and R_2 are equivalence relation the check whether $R_1 \cup R_2$ is an equivalence relation or not.

(ii) Find the number of positive integers not exceeding 440 that are either even of the square of an integer. 2+3

e) Let B be the set of all positive divisors of 48. Let us define $a + b = l \cdot c \cdot m$ of a and b, $a \cdot b = g \cdot c \cdot d$ of a and b, $a^{1} = 48/a$

Examine whether $(B, +, \cdot, ')$ is a Boolean algebra or not. 5

f) Let $(L, \prec)$ be a modular lattice and let $x \prec y, x, y, z \in L$ Then show that $x \wedge z = y \wedge z$ and $x \vee z = y \vee z \Rightarrow x = y$

3. Answer any ONE questions: 1x10 = 10

a) i) Define a relation P on $\mathbb{C}$ by "$(a + ib) P (c + id)$ if and only if $a \leq c$ and $b \leq d$" for $(a + ib), (c + id) \in \mathbb{C}$.

Show that P is a partial order relation.

ii) Obtain the principal disjunctive normal form of $\alpha = (\neg P V \neg Q) \neg (\neg P \wedge R)$

iii) Prove the following identity by induction $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\cdots+upto\, n\, terms = \frac{3}{4}-\frac{1}{2(n+1)}-\frac{1}{2(n+2)}$.

3+3+4

b) i) Show that every distributive lattice is modular.

ii) Among 100 students, 32 students study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology and 30 do not study any of the three subjects.

Find the number of students studying exactly one of the three subjects. Also find the number of students studying all the three subjects.

iii) Draw the Hasse diagram for the "less than or equal to" relation on $\{0,2,5,10,11,15\}$. 3+5+2
