Mathematics [Major]

[NEP]

B.Sc. First Semester End Examination-2024

(Regular & Supplementary Paper)

PAPER-MTMH MJ101

Full Marks: 60 *Time: 03 Hrs*

The figures in the right hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

Group - A

(Calculus: Marks-23)

1. Answer any FOUR questions: $4 \times 2 = 8$

(a) Evaluate $\lim_{x\to -1}(1-x^2)^{\frac{1}{\log(1-x)}}$

(b) Find the evolute of the parabola $y^2 = 4ax$

(c) Find the radius of curvature of the cycloid

$$x = a(t + sint), y = a(1 - cost)$$

(d) Find the envelope of the straight line $y = mx + \frac{a}{m}$, m being the variable parameter $(m \neq 0)$

(e) Find the length of the curve $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 8$

(f) Find if there is any point of inflection on the curve $y = 3 = 6(x-2)^5$.

2. Answer any ONE questions: **$1 \times 5 = 5$**

(a) Find the equation of the conic on which lie the eight points of intersection of the quadratic curve $xy(x^2 - y^2) + a^2y^2 + b^2x^2 - a^2b^2 = 0$ with its asymptotes.

(b) Establish a reduction formula for $\int \sin^m x \cos^n x \, dx$

3. Answer any one questions: $1 \times 10 = 10$

(a) (i)Show that for the ellipse, the radius of curvature at an extremity of the major axis is equal to half the latus rectum.

(ii) If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

(b) (i) Find the perimeter of the hypo-cycloid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$

(ii) Show that the envelopes of the circles $x^2 + y^2 - 2\alpha x - 2\beta y + \beta^2 = 0$, where α, β are parameters and whose centres lie on the parabola $y^2 = 4ax$ is $x(x^2 + y^2 - 2ax) = 0$ 5+5

Group-B

(Geometry: 23 Marks)

4. Answer any FOUR questions: **$2 \times 4 = 8$**

a) What does the equation $x^2 - 3xy + 3y^2 + 7x - 18y + 32 = 0$ become when the origin is moved to the point $(4,5)$ and the axes are turned through an angle $45°$?

) Determine the value of a and b so that the equation $4x^2 + 8xy + ay^2 + 2bx + 4y + 1 = 0$ represents a conic having infinitely many centers.

) Find the point on the conic $\frac{l}{r} = 1 - \cos\theta$ which has the smallest radius vector.

) Find the equation of the sphere which has the line segment joining the points $(2, 3, 4)$ and $(0, -1, 2)$as diameter in its standard.

Find the equation of a cone whose vertex is the origin, z-axis is axis of the cone with vertical angle $60°$.

Find the nature of the conicoid $2x^2 + 3y^2 - 8x + 6y - 12z + 11 = 0$

Answer any ONE question: **$5 \times 1 = 5$**

Reduce the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ to its standard form and show that it represents a hyperbola.

A sphere of radius $\frac{3}{2}$ units pass through the origin and meets the axes at A, B, C. Find the locus of the centroid of the triangle ABC.

nswer any ONE question: **10 × 1 = 10**

i) Find the polar equation of the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$, if the pole is at its right-hand focus and the positive direction of the $x-axis$ be the positive direction of the polar axis, also find the directrix equation in polar form.

ii) Show that the condition that the plane $ax+by+cz=0$ may cut the cone $xy+yz+zx=0$ in perpendicular lines is $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.

i) Find the necessary and sufficient condition that the plane $ax+by+cz=d$ is a tangent plane to the conicoid $lx^2+my^2+nz^2=p$.

i) Find the condition that two conics $\frac{l_1}{r}=1-e_1\cos\theta$ and $\frac{l_2}{r}=1-e_2\cos(\theta-\alpha)$ touch one another.

Group-C

(Linear Algebra: 14 Marks)

nswer any TWO questions: **2 × 2 = 4**

Prove that the set of vectors $\{(1,2,2),(2,1,2),(2,2,1)\}$ is linearly independent in $\mathbb{R}^3$.

(b) Use Cayley-Hamilton theorem to find A^{100} of the matrix $A=\begin{pmatrix}1&1\\0&1\end{pmatrix}$.

(c) Find all real x for which the rank of the matrix A is 2, where

$$A=\begin{pmatrix}1&2&3&1\\2&5&3&x\\1&1&6&x+1\end{pmatrix}.$$

8. Answer any TWO questions: **5 × 2 = 10**

(a) Find the value of k, for which the system of equations

$$kx+y+z=1$$
$$x+ky+z=1$$
$$x+y+kz=1$$

will have (i) unique solution, (ii) no solution and (iii) more than one solution.

(b) Show that the matrix $\begin{pmatrix}1&1&2\\2&4&4\\3&3&7\end{pmatrix}$ is non-singular and also find the invers of the given matrix using elementary row operations.

(c) Determine the normal form using congruence operation and obtain the rank and signature of the symmetric matrix

$$A=\begin{pmatrix}2&2&3\\2&5&6\\3&6&10\end{pmatrix}.$$
