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An application of real-coded genetic algorithm (RCGA) for mixed integer non-linear programming in two-storage multi-item inventory model with discount policy

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Abstract

The purpose of this research work is to solve mixed-integer non-linear programming problem with constraints by a real-coded genetic algorithm (RCGA). This GA is based on Roulette wheel selection, whole arithmetic crossover and non-uniform mutation. Here, mutation is carried out for the fine-tuning capabilities of the system by non-uniform operator whose action depends on the age of the population. This methodology has been applied in solving multiple price break structure and implemented for multi-item deterministic inventory control system having two separate storage facilities (owned and rented warehouse) due to limited capacity of the existing storage (owned warehouse). Also, demand rate is a linear function of selling price, time and non-linearly on the frequency of advertisement. The model is formulated with infinite replenishment and shortages are not allowed. The stocks of rented warehouse (RW) are transported to the owned warehouse (OW) in bulk-release rule. So, the mathematical model becomes a constrained non-linear mixed-integer problem. Our aim is to determine the optimal shipments, lot size of the two warehouses (OW and RW), shipment size and maximum profit by maximizing the profit function. The model is illustrated with numerical example and sensitivity analyses are performed with respect to different parameters.

Keywords: Inventory; Two-storage; All unit discount; Genetic algorithm; Bulk-release rule

1. Introduction

Generally, the basic assumption of classical inventory model is that the management purchases or produces a single item. However, in many real-life situations, this assumption is not correct. Instead of a single item, many companies or enterprises or retailers are motivated to store several items in their show-room for more profitable business affair. Another cause of their motivation is to attract the customers to purchase several items in one show room/shop. Multi-item classical inventory models under different resource constraints such

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as available floor space/shelf space, capital investment and average number of inventory, etc. are presented in the well-known books by Churchman and Ackoff [1], Silver and Peterson [2], etc., of this subject. Padmanabhan and Vrat [3] developed a multi-item multi-objective inventory model of deteriorating items with stock dependent demand by a non-linear goal programming method. Considering two constraints on available space and budget, Ben-Daya and Raouf [4] discussed a multi-item inventory model with stochastic demand. Abou-et-ata and Kotb [5] formulated and solved a multi-item inventory model with varying holding cost under two restrictions with the help of geometric programming. Recently, Guria et al. [6] studied multi-item EOQ model with storage facilities for uniform demand.

Now-a-days, the inventory systems with quantity discount are of growing interest due its practical importance in purchasing and material control. In the third world countries, with the introduction of open market system and advent of multi-nationals, there is a stiff competition amongst the companies to capture the maximum possible market. It is a common practice on the procurement in inventory systems that the suppliers (whole sellers) offers price discount to the retailers for purchase orders of large sizes. In general, there are two types of discount-All Unit Discount (AUD) and Incremental Quantity Discount (IQD). In AUD, the discount is available for every unit purchased where as in the incremental quantity discount system, the discount applies only to the additional units beyond the quantity over which the discount is given. Among these two types of discount, AUD is more popular and is usually utilized by the retailers.

The basic technique to solve the quantity discount models dates back to the early days of operational research. The basic model of EOQ under price breaks has been extensively analyzed in Hadley and Whitin [7] and reported in other books. Later, several authors have made extensions of the above model. Benton [8] considered quantity discount for MRP lot sizing, Majewicz and Swanson [9] for dynamic lot sizing, Goyal [10], Monahan [11], Kim and Hwang [12] for integrated decision making by supplier and buyer, Pirkul and Arkas [13] for multi-item inventory, Das [14] for generalized discount structure unifying the IQD and AUD policies. Also, Rubin et al. [15] proposed some computational simplifications and Das [16] presented a complete graphical solution to the discount problems. The purpose of the quantity discount is to offer a lower price which motivates retailers to increase order quantities and thereby reduce the total purchase cost. Therefore, quantity discount models always demand to buy a large number of items for which existing warehouse may not be sufficient to store these items. In the existing literature, it is found that the classical inventory models generally deal with a single storage facility. The basic assumption in these models is that the management has a storage with unlimited capacity. However, it is not true (e.g., in an important supermarket, the storage space of showroom is very limited) in the field of inventory management. Due to attractive price discount for bulk purchase or some problems in frequent procurement or very high demand of items, management decides to purchase a huge quantity of items at a time. These items cannot be stored in the existing storage (owned warehouse, OW) with limited capacities. So, for storing the excess items, one (some time more than one) warehouse is hired on rental basis. The rented warehouse RW is located near the OW or little away from it. Usually, the holding cost in RW is greater than the same in OW. Further, the items of RW are transported to OW in bulk fashion to meet the customer's demand until the stock level of RW is emptied.

In the last two decades, a good number of two warehouses inventory models have discussed by several researchers. This type of problem was first developed by Hartely [17] with the assumption of uniform demand of items. After Hartely [17], one may refer to the works of Sarma [18,19], Dave [20], Goswami and Choudhuri [21], Bhunia and Maiti [22,23], Pakkala and Achary [24], Benkherouf [25], Lee and Ma [26], Kar et al. [27] and others.

As an inventory problem is a decision-making problem which can be formulated as constrained/unconstrained non-linear optimization problem, there is a question: How it can be solved? Generally, most of the optimization problems of different inventory system are non-convex or non-concave optimization problems. In these problems, both local and global optimal solutions may exist. Then, special methods for global optimization are needed in order to solve these problems. Global optimization methods can be divided into deterministic and stochastic ones. Deterministic methods are usually based on some special assumptions on the problem to be solved, whereas stochastic methods utilize randomness. Because of their general nature, stochastic methods work even with discontinuous functions. Genetic algorithm (GA) represents this type of method. It is a robust technique, based on the natural selection and genetic production mechanism. It processes a group or population of possible solutions within a search space. This search is probability guided and stochastic, rather than deterministic or random searching which distinguish it from traditional methods. The basic idea behind the genetic algorithms is to artificially imitate the evaluation process of nature. The algorithms are based on the evaluation of a set of solutions, called a population. The population is upgraded by genetic operators in each iteration (generation). At each iteration (generation), the population consists of a number of individuals, i.e., possible solution of the problem. Typically, the population initialized by randomly generated individuals.

When individuals are encoded using real numbers the corresponding methods are called real-coded genetic algorithm. Each individual is a vector of variables where each variable is a real number. The suitability of an individual is determined by the value of a so-called fitness function based on the objective function. The population of next generation is created by these genetic operators: selection, crossover and mutation.

The selection operation chooses some offspring for survival according to their genetic diversity and fitness. The crossover operation generates offspring from two or more chosen individuals in the population by exchanging their genetic materials. The offspring thus inherit some characteristics from each parent. The mutation operation generates offspring by randomly changing one or several genes in an individual. Offspring may thus possess different characteristics from their parents. Mutation prevents local searches of the search space and increases the probability of finding global optima. Recently, GA has been successfully applied to a wide variety of problems such as Travelling salesman problems [28], Scheduling problems [29], Numerical Optimization [30], etc. Till now, only a very few researchers have applied it to solve the problem in the field of inventory control system. Among them, one may refer to the work of Khouja et al. [31], Sarkar and Charles [32], Mandal and Maiti [33], Pal et al. [34] among others.

In this research paper, we develop a multi-item two storage (OW and RW) multiple price breaks deterministic inventory model with a discount policy. The model is formulated as constrained non-linear mixed integer model and is solved by real-coded genetic algorithm with advanced GA operators. Also, demand is a function of selling price, time and frequency of advertisement. Shortages are not allowed, the stocks of RW are transported to OW in bulk-release fashion. Our objective is to determine the optimal shipments, lot-size of the OW and RW, shipment size and maximum profit by maximizing the profit function. Numerical examples illustrate that the above approaches are feasible and efficient.

2. Assumptions and notations

The following notations are used for the proposed model:

n number of items
W storage area or volume in RW
j any cycle of proposed inventory system ($j = 1, 2, \ldots$)

Parameters are used for the ith ($i = 1, 2, \ldots, n$) item in the jth cycle ($j = 1, 2, \ldots$)

$Q_{j,i}$ initial inventory units (decision variable)
$W_{j,i}$ storage capacity of OW
$m_{j.i}$ mark-up rate
w_i storage area or volume required for each item (m^2)
$T_{j,i}$ total time period
$P_{j,i}$ unit selling price
$p_{j,i}(=p_{j.i1}, p_{j,i2}, \ldots, p_{j,in})$ purchase cost per unit
L_1-system single storage/warehouse system
L_2-system two storage/warehouse system
$D_{j,i}$ demand rate
$N_{j,i}$ frequency of advertisement per replenishment
$H_{j,i}$, $F_{j,i}(F_{j,i} > H_{j,i})$ inventory carrying cost per unit per time in OW and RW, respectively
$\mu_{j,i}$ advertisement cost
n_i number of shipments from RW to OW (decision variable)

k_i units to be transported in each shipment
$t_{l,ji}$ consumption period of lth k_i units where $l = 1, 2, \ldots, n_i$
$b_{i1}, b_{i2}, \ldots, b_{in-1}$ 1st, 2nd, …, nth price breaks, respectively
$C_{3j,i}$ ordering cost per replenishment
$C_{tj,i}$ transportation cost per unit from RW to OW
$C_{Adj,i}$ total advertisement cost per replenishment
$C_{hj,i}$ total holding cost per cycle

The inventory model is developed under the following assumptions:

(i) The horizon of the inventory system is infinite.
(ii) Shortages are not allowed.
(iii) Lead time is zero.
(iv) The rate of replenishment is infinite.
(v) The selling price $P_{j,i}$ is determined by mark-up $m_{j,i}$ over the purchase cost $p_{j,i}$, i.e.,

$$P_{j,i} = m_{j,i} p_{j,i}. \tag{1}$$

(vi) If the lot-size $Q_{j,i}$ is less than the storage capacity of OW, the entire lot size is kept in OW. This type of inventory system is assumed as L_1-system. Otherwise, first OW is filled up completely and excess amount will be stored in RW. In this case, an additional transportation cost is incurred for special despatch of goods to RW. This system is known as L_2-system.
(vii) The storage capacity of OW (existing storage) is $W_{j,i}$ units and that of RW is limited.
(viii) The demand rate $D_{j,i}(P_{j,i}, t, N_{j,i})$ is dependent linearly on the unit selling price $P_{j,i}$, time t and non-linearly on frequency of advertisement, i.e.,

$$D_{j,i}(P_{j,i}, t, N_{j,i}) = (a_i - b_i P_{j,i} + c_i t) N_{j,i}^{\alpha_i}, \tag{2}$$

where a_i, b_i, c_i and α_i are non-negative constants.
(ix) The advertisement cost is $\mu_{j,i}$ $(0 < \mu_{j,i} < 1)$ fraction of the total selling price per jth cycle.
(x) The inventory holding cost $H_{j,i}$, $F_{j,i}(F_{j,i} > H_{j,i})$ are x_i and y_i percentages of the unit purchasing cost in OW and RW, respectively.
(xi) The purchasing multi-price break for AUD system is as follows:

$$p_{j,i} = \begin{cases} \$p_{j,i1}, & 0 < Q_{j,i} < b_{i1}, \\ \$p_{j,i2}, & b_{i1} \leqslant Q_{j,i} < b_{i2}, \\ \$p_{j,i3}, & b_{i2} \leqslant Q_{j,i} < b_{i3}, \\ \cdots & \\ \cdots & \\ \$p_{j,in}, & Q_{j,i} \geqslant b_{in-1}, \end{cases} \tag{3}$$

where $p_{j,i1} > p_{j,i2} \cdots > p_{j,in}$, $i = 1, 2, \ldots, n$.
(xii) The items of RW are transferred to OW in n_i shipments of which $k_i(k_i < W_{j,i})$ are to be transported in each shipment.
(xiii) The transportation cost of k_i units from RW to OW in each shipment is

$$a_i' + b_i'(k_i - S_i), \tag{4}$$

where $S_i(<k_i)$ is the maximum number of units which can be transported under a fixed charge a_i' and for every additional unit after S_i, a variable charge b_i' is to paid.
(xiv) $t_{l,ji}'$ $(l = 1, 2, \ldots, n_i)$ is the consumption period of the 1st lk_i units, i.e.,

$$t_{l,ji}' = \sum_{r=1}^{l} t_{r,ji}, \quad \text{where } t_{1,ji}' = t_{1,ji}.$$

Under the above assumptions two cases may arise:

Scenario 1: $Q_{j,i} > W_{j,i}$.
Scenario 2: $Q_{j,i} \leqslant W_{j,i}$.

Scenario 1: In this scenario, the purchase quantity $Q_{j,i}$ is greater than the capacity of the existing storage $W_{j,i}$. So, our problem becomes L_2-system, which is described and analyzed in the Section 3. Scenario 2: In this scenario, the purchase quantity $Q_{j,i}$ is less than/equal to the capacity of the existing storage $W_{j,i}$. As our model is two-storage inventory problem but this scenario violates the feasibility of the two-storage concept. So, we reject scenario 2.

3. Model description and analysis

Initially, a company purchases $Q_{j,i}$ units of ith item of which $W_{j,i}$ units are kept in OW and $(Q_{j,i} - W_{j,i})$ units are kept in RW. The stocks of OW are used to meet the customer's demand until the stock level of OW drops to $(W_{j,i} - k_i)$ units at the time of $t_{1,ji}$. At this stage, $k_i(k_i \leqslant W_{j,i})$ units are transported from RW to OW to restore the inventory into original level and to meet the further customer's demand. This process is continued until the stock of RW is fully exhausted. After the last shipment, only $W_{j,i}$ units are used to meet the customer's demand during the interval $(t_{n_i,ji}, T_{j,i})$. A pictorial representation of the system is given in Fig. 1.

Now, our problem is to determine the optimal values of n_i, $Q_{j,i}$ such that the average profit for this model is maximized and also to determine the corresponding values of k_i and $T_{j,i}$.

The total demand during the planning horizon $(0, T_{j,i})$ is $Q_{j,i}$ then

$$Q_{j,i} = \int_0^{T_{j,i}} D_{j,i}\,\mathrm{d}t = \int_0^{T_{j,i}} (a_i - b_iP_{j,i} + c_it)N_{j,i}^{\alpha_i}\,\mathrm{d}t \;\Rightarrow\; N_{j,i}^{\alpha_i}c_iT_{j,i}^2 + 2N_{j,i}^{\alpha_i}(a_i - b_iP_{j,i})T_{j,i} - 2Q_{j,i} = 0, \tag{5}$$

which corresponds the feasible solution

$$T_{j,i} = \frac{-N_{j,i}^{\alpha_i}(a_i - b_iP_{j,i}) + \sqrt{N_{j,i}^{2\alpha_i}(a_i - b_iP_{j,i})^2 + 2Q_{j,i}N_{j,i}^{\alpha_i}}}{c_iN_{j,i}^{\alpha_i}}. \tag{6}$$

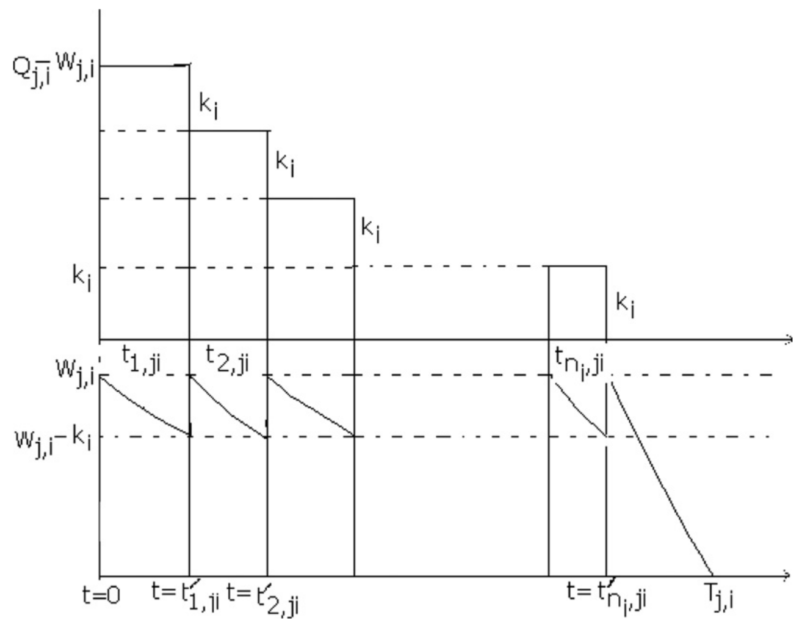

Fig. 1.

The demand in the interval $(0, t'_{l,ji})$ is lk_i $(l = 1, 2, \ldots, n_i)$ then

$$lk_i = \int_0^{t'_{l,ji}} (a_i - b_i P_{j,i} + c_i t) N_{j,i}^{\alpha_i} \, \mathrm{d}t,$$

which implies

$$N_{j,i}^{\alpha_i} c_i t'^2_{l,ji} + 2N_{j,i}^{\alpha_i}(a_i - b_i P_{j,i}) t'_{l,ji} - 2lk_i = 0, \tag{7}$$

which gives only the real solution

$$t'_{l,ji} = \frac{-N_{j,i}^{\alpha_i}(a_i - b_i P_{j,i}) + \sqrt{N_{j,i}^{2\alpha_i}(a_i - b_i P_{j,i})^2 + 2lc_i k_i N_{j,i}^{\alpha_i}}}{c_i N_{j,i}^{\alpha_i}} \quad \text{for } l = 1, 2, \ldots, n_i. \tag{8}$$

Thus,

$$t_{l,ji} = t'_{l,ji} - t'_{l-1,ji} = \frac{\sqrt{N_{j,i}^{2\alpha_i}(a_i - b_i P_{j,i})^2 + 2lc_i k_i N_{j,i}^{\alpha_i}} - \sqrt{N_{j,i}^{2\alpha_i}(a_i - b_i P_{j,i})^2 + 2(l-1)c_i k_i N_{j,i}^{\alpha_i}}}{c_i N_{j,i}}. \tag{9}$$

Again the relation between $Q_{j,i}$ and k_i as

$$\begin{aligned} Q_{j,i} &= n_i k_i + W_{j,i}, \\ k_i &= (Q_{j,i} - W_{j,i})/n_i. \end{aligned} \tag{10}$$

4. The inventory cost function

The total cost in the jth cycle consists of the following components:

(a) Transportation cost ($C_{tj,i}$), (b) Advertisement cost ($C_{Adj,i}$), (c) Holding cost ($C_{hj,i}$), (d) Purchase cost ($p_{j,i}$), (e) Set-up cost ($C_{3j,i}$).

(a) *Transportation cost*: The transportation cost for transferred the items/goods from RW to OW in n_i shipments is given by

$$\begin{aligned} C_{tj,i} &= n_i[a'_i + b'_i(k_i - S_i)] \quad \text{for } k_i > S_i, \\ &= n_i a'_i, \quad \text{otherwise}. \end{aligned} \tag{11}$$

(b) *Advertisement cost*: The total advertisement cost per replenishment is

$$C_{Adj,i} = \mu_{j,i} Q_{j,i} P_{j,i} N_{j,i}. \tag{12}$$

(c) *Holding cost*: The inventory time units in RW is

$$\sum_{l=1}^{n_i} [Q_{j,i} - W_{j,i} - (l-1)k_i] t_{l,ji}.$$

The holding cost of the items in RW in the jth cycle is

$$F_{j,i} \sum_{l=1}^{n_i} [Q_{j,i} - W_{j,i} - (l-1)k_i] t_{l,ji}. \tag{13}$$

Between $(l-1)$th and lth $(l = 1, 2, \ldots, n_i)$ shipments, i.e., during the interval $(t'_{l-1,ji}, t'_{l,ji})$, k_i units in OW are used to meet the customer's demand for the period $t_{l,ji}$.

So the holding cost $E_{l,ji}$ for these items in OW are given by

$$E_{l,ji} = H_{j,i} \int_{t'_{l-1,ji}}^{t'_{l,ji}} (t - t'_{l-1,ji}) D_{j,i} \, \mathrm{d}t. \tag{14}$$

The total of all such holding cost is

$$\sum_{l=1}^{n_i} E_{l,ji} = N_{j,i}^{\alpha_i} H_{j,i} \sum_{l=1}^{n_i} \{(a_i - b_i P_{j,i} - c_i t'_{l-1,ji})(t'^2_{l,ji} - t'^2_{l-1,ji})/2 + c_i(t'^3_{l,ji} - t'^3_{l-1,ji})/3 - (t'_{l,ji} - t'_{l-1,ji})t'_{l-1,ji}(a_i - b_i P_{j,i})\}. \tag{15}$$

As the quantity $(W_{j,i} - k_i)$ units is kept unused in OW for a period $t'_{n_i,ji}$, the holding cost for these quantities is $H_{j,i}(W_{j,i} - k_i)t'_{n_i,ji}$.

When the last shipment, i.e., n_ith k_i units arrives in OW, the on hand inventory in OW becomes $W_{j,i}$ which is cleared during the interval $(t'_{n_i,ji}, T_{j,i})$.

The holding cost for these units is

$$H_{j,i} \int_{t'_{n_i,ji}}^{T_{j,i}} (t - t'_{n_i,ji}) D_{j,i}\, \mathrm{d}t = N_{j,i}^{\alpha_i} H_{j,i} \sum_{l=1}^{n_i} \{(a_i - b_i P_{j,i} - c_i t'_{n_i,ji})(T^2_{j,i} - t'^2_{n_i,ji})/2 + c_i(T^3_{j,i} - t'^3_{n_i,ji})/3 - (T_{j,i} - t'_{n_i,ji})t'_{n_i,ji}(a_i - b_i P_{j,i})\}. \tag{16}$$

The total holding cost $(C_{hj,i})$ is the sum of equations (13)–(16).

Thus, the profit function is

$$\max Z = \langle \text{Sales revenue} \rangle - \langle \text{Purchase cost} \rangle - \langle \text{Holding cost} \rangle - \langle \text{Set-up cost} \rangle - \langle \text{Transportation cost} \rangle - \langle \text{Advertisement cost} \rangle.$$

Now, the average profit for the two-storage system over $(0, T_{j,i})$ is given by

$$\max\ Z(Q, n) = \max\ \sum_{i=1}^{n} Z_i(Q_{j,i}, n_i) = \sum_{i=1}^{n} [Q_{j,i}P_{j,i} - Q_{j,i}p_{j,i} - C_{hj,i} - C_{3j,i} - C_{tj,i} - C_{ADj,i}]/T_{j,i}. \tag{17}$$

The above profit function Z_i is a function of two variables $Q_{j,i}$ and n_i of which n_i is discrete variables and $Q_{j,i}$ is continuous variable. Our objective is to determine the values of continuous and discrete variables by maximizing the profit function $Z_i(Q_{j,i}, n_i)$.

Hence, our problem is

$$\begin{aligned} &\max \quad \sum_{i=1}^{n} Z_i(Q_{j,i}, n_i) \\ &\text{s.t.} \quad \sum_{i=1}^{n} w_i(Q_{j,i} - W_{j,i}) \leqslant W. \end{aligned} \tag{18}$$

The above problem (18) is a non-linear mixed integer maximization problem. It is a formidable task to prove that the problem (18) is either concave or not. Hence, to find out the global solution, one can use any well-known soft computing method. In this paper, we shall use real-coded GA to find out the global solution (best found solution).

5. Genetic algorithm

Genetic algorithms are heuristic search process for optimization that resembles natural selection. In most cases, they can find the global optimum solution with a high probability. They mimic the process of natural selection and is based on Darwin's survival of the fittest principles. In this algorithm, a population of individuals (potential solutions) undergo a sequence of unary (mutation type) and higher order (crossover type) transformations. These individuals select the next generation. This new generation contains a higher proportion of the characteristics possessed by the 'good' members of the previous generation and in this way good characteristics are spread over the population and mixed with other good characteristics. After a few numbers of generations, the program either converges or is terminated and the best individual is taken as the optimal solution. It is generally accepted that any Genetic Algorithm to solve a problem must have the following basic components:

- Values of parameters (population size, probabilities of applying genetic operators, etc.) of Genetic Algorithms.
- Chromosome representation.
- Initial population production.
- Evaluation function rating solutions in terms of their fitness.
- Selection process.
- Genetic operators (crossover and mutation) that alter the genetic composition of parents during reproduction.

The coding of real-coded Genetic Algorithm (RCGA) is shown in Fig. 2.

5.1. Parameters of GA

Now, we have to set the different parameters on which GA depends. All these parameters are the population size (POPSIZE), probability of crossover (PCROS), probability of mutation (PMUTE), maximum number of generation (MAXGEN). About the population size of GA, there is no clear indication how large it should be. If the population is too large, there arises some difficulties in storing of the data. But if the population size is too small, there may not be enough populations for good crossover. In our present study, we have taken the following values of parameters:

$$\text{POPSIZE} = 100, \quad \text{PCROS} = 0.8, \quad \text{PMUTE} = 0.1, \quad \text{MAXGEN} = 15.$$

5.2. Chromosome representation

A main problem in applying a GA is to design an appropriate chromosome representation of solutions of the problem with genetic operators. Traditional binary vectors used to represent the chromosome are not effective in many highly physical non-linear problems. As our proposed model is highly non-linear containing two different types of variables (discrete and continuous); to mitigate this difficulty, a real number representation is used here. A real row matrix $V_{j,i} = [V_{j,i1}, V_{j,i2}]$ is used to represent a chromosome where $V_{j,i1}$ and $V_{j,i2}$ represent $Q_{j,i}$ and n_i $(i = 1, 2, \ldots, n)$ and $(j = 1, 2, \ldots, \text{POPSIZE})$, respectively.

```
begin
 Initialize the parameters of GA
  t←0 [ t represents the number of current generation ]
  initialize p(t) [ p(t) represents the population at t th generation ]
  evaluate p(t)
    while ( not terminate condition )
     {
        t ←t +1
        select p(t) from p( t-1)
        alter ( crossover and mutate ) p(t)
        evaluate p(t)
        upgrade the result, if possible
     }
print the best found result
end
```

Fig. 2. Coding of RCGA.

5.3. Initialization

Here, the parameter POPSIZE denotes the number of chromosomes and these chromosomes are initialized randomly. Usually, it is difficult for complex optimization problems to produce feasible chromosome explicitly. The population generation techniques proposed in our present model are as follows:

> In GA, POPSIZE number of chromosomes $V_{1,i}, V_{2,i}, \ldots, V_{\text{POPSIZE},i}$ are generated randomly within the boundary of the component where each solution satisfies the resource constraints of the problem. This set of chromosomes is taken as initial population.

5.4. Evaluation function

To evaluate the value of the objective function $Z(Q,n)$ due to the potential solution $V_{j,i} = [V_{j,i1}, V_{j,i2}]$ where $(j = 1,2,\ldots,\text{POPSIZE})$ and $(i = 1,2,\ldots,n)$, purchase cost of $Q_{j,i}$ is taken as $p_{j,i1}$ per unit if $0 < Q_{j,i} < b_{i1}, p_{j,i2}$ per unit if $b_{i1} \leqslant Q_{j,i} < b_{i2}$ and so on for the nth price break $p_{j,in}$ per unit if $Q_{j,i} \geqslant b_{in-1}$ for $i = 1,2,\ldots,n$ [c.f. Eq. (3)]. Following this, holding costs, advertisement cost, revenue and time period for the items are also calculated. The value of the objective (profit) function due to the chromosome $V_{j,i}$ is taken as fitness value of $V_{j,i}$ and it is denoted by eval$(V_{j,i})$.

5.5. Selection

The purpose of selection is, of course, to emphasize the better individuals in the population for recombination in hopes that their offspring will in turn have even higher fitness. Selection has to be balanced with variation from crossover and mutation: too strong selection means that sub-optimal highly fit individuals will take over the population, by reducing the diversity needed for further change and progress; too weak selection will result in too-slow evolution. Here, we adopt well-known Roulette Wheel scheme. The selection process is as follows:

(i) Find the total fitness of the population $F = \sum_{j=1}^{\text{POPSIZE}} \text{eval}(V_{j,i})$.
(ii) Find the probability p_j of selection for each chromosome $V_{j,i}$ by the formula $p_j = \text{eval}(V_{j,i})/F$.
(iii) Find cumulative probability q_j for each chromosome $V_{j,i}$ using the formula

$$q_j = \sum_{i-1}^{j} p_i.$$

(iv) Generate a random number r in $[0, 1]$.
(v) If $r < q_1$ then select the first chromosome $V_{1,i}$ otherwise select the jth chromosome $V_{j,i}(2 \leqslant j \leqslant \text{POPSIZE})$ such that $q_{j-1} < r \leqslant q_j$.
(vi) Repeat (iv) and (v) POPSIZE times and obtain POPSIZE numbers of chromosomes.

5.6. Crossover operation

The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. We define a parameter PCROS of a genetic system as the probability of crossover. Then the probability gives us the expected number PCROS * POPSIZE of chromosomes which undergoes the crossover operation. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection of chromosomes for new population, crossover, namely arithmetic crossover operation is used and is defined as a linear combination of two consecutive selected chromosomes. In our problem, each chromosome $V_{j,i}$ has two genes $V_{j,i1}$ and $V_{j,i2}$ which represent continuous and discrete variables, respectively. Different steps of crossover operation are given below:

(i) Find the integral value of PCROS * POPSIZE and store it in N.
(ii) Generate a random real number r in $(0,1)$.
(iii) Select two consecutive chromosomes $V_{l,i1}$ and $V_{k,i2}$ randomly among population for crossover if $r <$ PCROS.
(iv) Generate a random number c in the interval $(0,1)$.
(v) Produce the offsprings X and Y as

$$X = c \cdot V_{l,i1} + (1-c) \cdot V_{k,i2},$$
$$Y = c \cdot V_{k,i2} + (1-c) \cdot V_{l,i1}.$$

(vi) Select two integer chromosomes $V_{l,i2}$ and $V_{k,i2}$. If $V_{l,i2} > V_{k,i2}$ and generate a random integer number g between 0 and $V_{l,i2} - V_{k,i2}$. Then produce $X' = V_{k,i2} + g$ and $Y' = V_{l,i2} - g$.
(vii) Repeat steps (i)–(vi) for $N/2$ times.

If the feasible set is convex, this arithmetical crossover operation ensures that both children are feasible if both parents are. However, in many cases, the feasible set is not necessarily convex or hard to verify convexity. So we must check the feasibility to each child. If both children are feasible, then we replace the parents by them. If not, we keep the feasible one if it exists, and then re-do the crossover operator by generating the random number c until two feasible children are obtained or a given number of cycles is finished.

5.7. Mutation operation

This operation is responsible for fine tuning capabilities of the system. it is applied to a single chromosome. Here, we shall use uniform and non-uniform mutations for discrete and continuous variable, respectively. The action of non-uniform mutation depends on the age of the population. If the chromosome $V_{k,i}$ is selected for this mutation and domain of $V_{k,i}$ is $[l_{k,i}, u_{k,i}]$, then the reduced value of $V_{k,i}$ is given by

$$V_{k,i} = \begin{cases} V_{k,i} + \Delta(t, u_{k,i} - V_{k,i}) & \text{if a random digit is 0,} \\ V_{ik} - \Delta(t, V_{k,i} - l_{k,i}) & \text{if a random digit is 1.} \end{cases}$$

where $k \in [1,2]$ and $\Delta(t,y)$ returns a value in the range $[0,y]$.

In our study, we have taken

$$\begin{aligned} \Delta(t,y) &= \text{a random integer between } [0,y] \quad \text{for discrete variable} \\ &= y[1 - r^{(1-t/T)b}] \quad \text{for a continuous variable,} \end{aligned}$$

where r is a random number in $[0,1]$, $T =$ MAXGEN, $t =$ represents the current generation and b is the constant.

6. The algorithm of AUD for multi-price break

The quantity $Q^*_{j,i}$ which maximizes the average total profit for the above model can be determined by the following procedure which is similar to that proposed by Wee [35]:

Step 1. Starting with the lowest unit cost (in our case $p_{j,in}$),
  Step 1.1. calculate the optimal n_i and $Q_{j,i}$ values using the proposed procedure;
  Step 1.2. calculate the respective k_i from Eq. (9) and check if $Q_{j,i}$ is valid quantity;
  Step 1.3. continue until the first valid ordered quantity, $Q_{j,i}$ is obtained and calculate the optimum $Z(Q_{j,i}, n_i)$.

Step 2. Calculate the optimal net profit for all price break quantities larger than $Q_{j,i}$ say $Z_{bik+1}, \ldots, Z_{bin-1}$.

Step 3. Compare $Z(Q_{j,i}, n_i)$, $Z_{bik+1}, \ldots, Z_{bin-1}$ and select $Q^*_{j,i}$, n^*_i and k^*_i corresponding to the maximum of these values.

7. Numerical examples

To illustrate the developed model, an example has been considered. Though the values of the model parameters have not been selected from any case study, the values considered here are feasible. Here, we have considered two items and four-price break. The input values are given in Table 1 and optimum results are displayed in Table 2.

Table 1
Different parametric values for proposed model (input data)

i	a_i	b_i	c_i	$W_{j,i}$ (Units)	S_i (Units)	$m_{j,i}$	x_i (%)	y_i (%)	$\mu_{j,i}$	$C_{3j,i}$ ($)	α_i	$N_{j,i}$	a'_i ($)	b'_i ($)
1	120	0.4	5	75	10	1.5	18	12	0.02	15	0.5	2	13	0.4
2	110	0.6	4	35	12	1.6	20	15	0.03	13	0.3	2	15	0.5

$w_1 = 2.5\ \text{m}^2,\ w_2 = 2\ \text{m}^2,\ W = 160\ \text{m}^2$.

Table 2
Optimum results for proposed model

i	$Q^*_{j,i}$	n^*_i	$T^*_{j,i}$	k^*_i	Z^* ($)
1	112.3776	3	0.6983	12.4592	827.8730
2	58.7623	2		11.8812	

Table 3
Sensitivity analysis of the proposed model

Changing of parameters		i	−20%	−10%	−5%	5%	10%	20%
$W_{j,i}$	Δn_i	1	Infeasible	Infeasible	Infeasible	−33.33	33.33	0
		2	Infeasible	Infeasible	Infeasible	0	0	0
	ΔZ		Infeasible	Infeasible	Infeasible	−6.2596	−6.5332	−3.9502
	Δk_i	1	Infeasible	Infeasible	Infeasible	99.5674	−11.2933	30.8805
		2	Infeasible	Infeasible	Infeasible	−30.5289	−30.1957	−26.6088
	$\Delta T_{j,i}$		Infeasible	Infeasible	Infeasible	14.0806	12.5352	23.1800
$m_{j,i}$	Δn_i	1	Infeasible	0	0	0	0	0
		2	Infeasible	0	0	0	0	0
	ΔZ		Infeasible	−53.8234	−26.8181	26.6315	53.0762	105.4056
	Δk_i	1	Infeasible	0.0000	0.0000	0.0000	0.0000	0.0000
		2	Infeasible	−0.0002	−0.0002	−0.0002	−0.0002	−0.0002
	$\Delta T_{j,i}$		Infeasible	−0.6842	−0.3436	0.3445	0.6920	1.3943
a_i	Δn_i	1	0	0	0	0	0	0
		2	0	0	0	0	0	0
	ΔZ		−19.0684	−9.1599	−4.4853	4.2949	8.3978	16.0239
	Δk_i	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	−0.0002	−0.0002	−0.0002	−0.0002	−0.0002	−0.0002
	$\Delta T_{j,i}$		26.0869	11.5662	5.4718	−4.9408	−9.4215	−17.2370
b_i	Δn_i	1	0	0	0	0	0	0
		2	0	0	0	0	0	0
	ΔZ		1.6836	0.8418	0.4209	−0.4209	−0.8418	−1.6839
	Δk_i	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	−0.0002	−0.0002	−0.0002	−0.0002	−0.0002	−0.0002
	$\Delta T_{j,i}$		−1.3585	−0.6842	−0.3436	0.3445	0.6920	1.3943
c_i	Δn_i	1	0	0	0	0	0	0
		2	0	0	0	0	0	0
	ΔZ		−0.3379	−0.1687	−0.0842	0.0842	0.1682	0.3360
	Δk_i	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	−0.0002	−0.0002	−0.0002	−0.0002	−0.0002	−0.0002
	$\Delta T_{j,i}$		0.3033	0.1508	0.0749	−0.0762	−0.1514	−0.3011

For the 1st item For the 2nd item

$$p_{j,1} = \begin{cases} \$15.00, & 0 < Q_{j,1} < 50, \\ \$14.00, & 50 \leqslant Q_{j,1} < 75, \\ \$13.25, & 75 \leqslant Q_{j,1} < 150, \\ .\$12.00, & Q_{j,1} \geqslant 150. \end{cases} \quad p_{j,2} = \begin{cases} \$12.00, & 0 < Q_{j,2} < 25, \\ \$11.25, & 25 \leqslant Q_{j,2} < 70, \\ \$10.50 & 70 \leqslant Q_{j,2} < 100, \\ \$9.75, & Q_{j,2} \geqslant 100. \end{cases}$$

8. Sensitivity analysis

The earlier numerical example is used to study the effect of under or overestimation of various parameters on optimal cycle length, maximum net profit, optimal number of shipments from RW to OW and optimal units transported in each shipment of the inventory system. Here, we employ $\Delta T_{j,i} = (T'_{j,i} - T_{j,i})/T_{j,i} \times 100\%$, $\Delta Z = (Z' - Z)/Z \times 100\%$, $\Delta n_i = (n'_i - n_i)/n_i \times 100\%$ and $\Delta k_i = (k'_i - k_i)/k_i \times 100\%$ as a measure of sensitivity, where n_i, k_i, Z and $T_{j,i}$ are the true values and n'_i, k'_i, Z' and $T'_{j,i}$ the estimated values. The sensitivity analysis is shown by increasing or decreasing the parameters by 5%, 10% and 20%, taking one or more at a time and keeping the others at their true values. The results are presented in Table 3, which are self-explanatory.

9. Conclusion

Here, for the first time, a multi-item two-storage inventory model with AUD has been formulated with a resource constraint and successfully solved by real-coded GA with crossover and mutation for integer and non-integer variables, developed for this purpose. Due to the complexities, till now, none has attempted this type of multi-item discounted problem with space constraint by conventional price-break methodology. Though the model has been illustrated with only four price breaks, the GA developed here can be easily extended to include more than four price-break points. Similarly, the present methodology can be easily applied to the inventory problems with several resource constraints like limitation on budgetary costs, etc. in addition to the space constraint illustrated in this paper. Moreover, the present GA can also applied to other types of inventory models with variable demand, fixed time horizon, etc. along with quantity discount formulated in fuzzy, probabilistic or fuzzy-probabilistic environments.

References

[1] C.W. Churchman, E.L. Ackoff, Introduction to Operations Research, John Wiley, New York, 1957.

[2] E.A. Silver, R. Peterson, Decision System for Inventory Management and Research, John Wiley, New York, 1985.

[3] G. Padmanabhan, P. Vrat, Analysis of multi-item inventory system under resource constraint: a non-linear goal programming approach, Engineering Cost and Production Management 13 (1990) 104–112.

[4] M. Ben-Daya, A. Raouf, On the constrained multi-item single-period inventory problem, International Journal of Operation Production Management 13 (1993) 104–112.

[5] O.M. Abou-el-ata, K.A.M. Kotb, Multi-item inventory model with varying holding costs under two restrictions: a geometric approach, Production Planning and Control 8 (1997) 608–611.

[6] S.N. Guria, D. Maiti, A.K. Bhunia, On the multi-item EOQ models with two storage facilities, Modelling Measurement and Control 21 (2000) 31–40.

[7] G. Hadley, T.M. Whitin, Analysis of Inventory Systems, Prentice-Hall, Englewood Cliffs, NJ, 1963.

[8] W.C. Benton, Multiple price breaks and alternative purchase lot-sizing procedures in material requirements planning systems, International Journal of Production Research 23 (1985) 1025–1047.

[9] D. Majewicz, L.A. Swanson, Inventory ordering and quantities discounts with time varying demands: a programming application, Production and Inventory Management 19 (1978) 91–102.

[10] S.K. Goyal, An integrated inventory model for a single supplier single customer problem, International Journal of Production Research 15 (1977) 107–111.

[11] J.P. Monahan, A quantity discount pricing model to increase vendor profits, Management Science 30 (1984) 720–726.

[12] K.H. Kim, H. Hwang, An incremental discount schedule with multiple customers and single price break, European Journal of Operational Research 35 (1988) 71–79.

[13] H. Pirkul, O.A. Arkas, Capacited multiple item ordering problem with quantity discounts, IIE Transactions 17 (1985) 206–211.

[14] C. Das, A graphical approach to price-break analysis – 3, Journal of Operational Research Society 35 (1985) 995–1001.

[15] P.A. Rubin, D.M. Dilts, B.A. Barron, Economic order quantities with quantity discount: grandma dos it best, Decision Science 14 (1983) 270–281.
[16] C. Das, A generalized discount structure and some dominance rules for selecting the price break EOQ, European Journal of Operational Research 34 (1988) 27–38.
[17] R.V. Hartely, Operations Research – A Managerial Emphasis, Goodyear Publishing Company, 1976, pp. 315–317.
[18] K.V.S. Sarma, A deterministic order-level inventory model for deteriorating items with two storage facilities, European Journal of Operational Research 29 (1987) 70–72.
[19] K.V.S. Sarma, A deterministic inventory model with two levels of storage and an optimum release rule, Opsearch 29 (1983) 175–180.
[20] U. Dave, On the EOQ models with two levels of storage, Opsearch 25 (1988) 190–196.
[21] A. Goswami, K.S. Chaudhuri, An economic order quantity model for items with two levels of storage for a linear trend in demand, Journal of Operational Research Society 43 (1992) 157–167.
[22] A.K. Bhunia, M. Maiti, A two warehouses inventory model for deteriorating items with linear trend in demand and shortages, Journal of Operational Research Society 49 (1997) 287–292.
[23] A.K. Bhunia, M. Maiti, A two-warehouse inventory model for a linear trend in demand, Opsearch 31 (1994) 318–329.
[24] T.P.M. Pakkala, K.K. Achary, A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate, European Journal of Operational Research 57 (1992) 71–76.
[25] L.A. Benkherouf, A deterministic order level inventory model for deteriorating items with two storage facilities, International Journal of Production Economics 48 (1997) 167–175.
[26] C.C. Lee, C.Y. Ma, Optimal inventory policy for deteriorating items with two warehouse and time dependent demands, Production Planning and Control 7 (2000) 689–696.
[27] S. Kar, A.K. Bhunia, M. Maiti, Deterministic inventory model with two levels of storage, a trend in demand and a fixed time horizon, Computers and Operations Research 28 (2001) 1315–1331.
[28] S. Forest, Genetic algorithms for multi-objective optimizations, formulation, discussion and generalization, in: Proceedings of 5th International Conference on Genetic Algorithms, Margen Kaufmanns, California, 1993, pp. 416–423.
[29] L. Davis, Handbook of Genetic Algorithms, Van Nostrand Reinhold, New York, 1991.
[30] Z. Michalawicz, Genetic Algorithms + Data Structures = Evaluation Programs, Springer, Berlin, 1992.
[31] M. Khouja, Z. Michalawicz, M. Wilmot, The use of genetic algorithms to solve the economic lot-size scheduling problem, European Journal of Operational Research 110 (1998) 509–524.
[32] R. Sarkar, Charles Newton, A genetic algorithm for solving economic lot-size scheduling problem, Computers and Industrial Engineering 42 (2002) 189–198.
[33] S. Mandal, M. Maiti, Multi-item fuzzy EOQ models using genetic algorithm, Computers and Industrial Engineering 44 (2002) 105–117.
[34] P. Pal, A.K. Bhunia, An application of real-coded genetic algorithm (for mixed integer non-linear programming in an optimal two-warehouse inventory policy for deteriorating items with a linear trend in demand and fixed planning horizon), International Journal of Computer Mathematics 82 (2005) 163–175.
[35] H.M. Wee, Deteriorating inventory model with quantity discount, pricing and partial backordering, International Journal of Production Economics 59 (1999) 511–518.