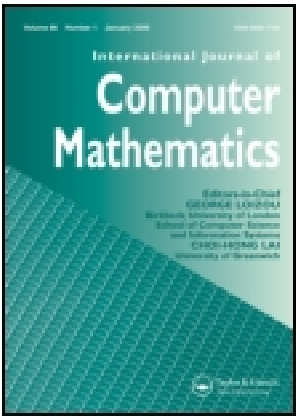

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Discounted multi-item inventory model via genetic algorithm with Roulette wheel selection, arithmetic crossover and uniform mutation in constraints bounded domains

A. K. Maiti [a] & M. Maiti [a]
[a] Department of Mathematics, Vidyasagar University, Midnapore, WB, India

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A.K. Maiti* and M. Maiti

Department of Mathematics, Vidyasagar University, Midnapore, WB, India

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Genetic Algorithm (GA) with different logic structures for price breaks has been developed and implemented for a multi-item inventory control system of breakable items like the items made of glass, mud, porcelain, etc. with all unit discount (AUD), incremental quantity discount (IQD) and a combination of these discounts. Here, AUD and IQD on purchasing price with two price breaks are allowed. Also, demand and breakability of the items are stock-dependent. Shortages are not allowed. Replenishment is instantaneous. Selling price is a mark-up of the purchasing cost. For storage, warehouse capacity is limited. For the present model, GA has been developed in real code representation using Roulette wheel selection, arithmetic crossover and uniform mutation. This algorithm has been implemented successfully to find the optimum order quantities for the above inventory control system to achieve the maximum possible profit. The algorithm and the inventory model have been illustrated numerically and some sensitivity analyses with respect to breakability and demand are presented.

Keywords: inventory; breakability; all unit discount; incremental quantity discount; genetic algorithm

2000 AMS Subject Classification: 90B05

1. Introduction

In the classical deterministic inventory models different types of demand are considered e.g., constant, dependent on time, selling price, stock, etc., It has been acknowledged that displayed inventory has an effect on sale for many retailers. This means that the demand rates of these items may be dependent on displayed stock level. This type of demand in different forms was considered by Baker and Urban [4], Mandal and Phaujder [27], Datta and Pal [9, 10], Urban [35], Mandal and Maiti [25, 26], Zhou [38, 39], Maiti and Maiti [19, 23] and others.

Generally, classical inventory models are developed under the basic assumption that the management purchases or produces a single item. However, in many real-life situations, this assumption is not correct. Instead of a single item, many companies or enterprises or retailers are motivated to store several items in their showrooms for more profitable business affairs. Another cause of their motivation is to attract the customers to purchase several items in one showroom/shop. Multi-item classical inventory models under different resource constraints such

*Corresponding author. Email: ajoy_maiti2003@yahoo.co.in

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as available floor space/shelf space, capital investment and average number of inventory, etc., are presented in the well-known books by Churchman [8], Silver and Peterson [33] and others. Padmanabhan and Vrat [30] developed a multi-item multi-objective inventory model of deteriorating items with stock dependent demand by a goal programming method. Considering two constraints on available space and budget, Ben-Daya and Raouf [5] discussed a multi-item inventory model with stochastic demand. Recently, Guria *et al.* [16] studied multi-item EOQ model with two-storage facilities for uniform demand. Kar *et al.* [18] solved two-shop inventory model of deteriorating multi-items with constraints on space and investment. Normally, decision-making problems are formulated as unconstrained/constrained nonlinear optimization problem, which are solved by traditional direct and gradient-based optimization method. Among the limitations of these methods, one is that the traditional nonlinear optimization methods very often stuck to the local optimum. To overcome some of these limitations, a soft computing method, genetic algorithm (GA) is very popular. GAs have been applied in different areas like neural networks (c.f. Pal *et al.* [31]), scheduling (Davis [11]), numerical optimization (Michalewicz [28]), pattern recognition (Gelsema [14]), etc. There are very few papers where GAs have been applied in the field of inventory control system [32]. Disney *et al.* [12] solved classical inventory control systems using GA optimization, Mondal and Maiti [29] developed a multi-item fuzzy EOQ model using GA, Gaafar [13] studied GA to dynamic lot-sizing with batch ordering, Altiparmak *et al.* [3] solved multi-objective supply chain networks via GA, Smith [34] applied GA on multiple inventory problem. Recently, Maiti and Maiti [19] did work on damageable items in imperfect production process via GA. Also Maiti and Maiti [22, 23] developed two-warehouse problem in fuzzy environment, Maiti *et al.* [20, 21] applied GA respectively in two-warehouse inventory model with discount and with random planning horizon.

In inventory, there is a number of research papers dealing with deteriorating items. But, the inventory of breakable goods, that is the items made of glass, ceramic, china clay, etc., is different from that of deteriorating items and did not get much attention from the O.R. scientists. Recently Mandal and Maiti [24–26] did some works on these items. These items break more in numbers as the stock increases.

Again, quantity discount is of growing interest due to its practical importance in purchasing and control of an item. Normally, one derives the better marginal cost of purchase/production availing the opportunities of cost savings through bulk purchase/production. Therefore, a retailer reacts to more stock during the period of discount offered from several sources. Now-a-days, in the third-world countries, with the introduction of the open market system and the advent of multi-nationals, there is stiff competition among the companies to win over the maximum possible market. They allure the customers by giving quantity discounts in different forms. The popular discounts are all unit discount (AUD) and incremental quantity discount (IQD). Under AUD, the unit price of 'all units' decreases as the order size increases according to the price schedule offered by the supplier. In AUD, the discounts are offered to every unit purchased whereas in IQD, discounts are offered only to the additional units ordered beyond a specified quantity over which the discount is given.

In the literature of discounted inventory problems, Benton [6] considered an inventory system with quantity discount for multiple price breaks and alternative purchase lotsizing policy, Abad [1, 2] for selling price and lot size when suppliers offer AUD and IQD, Chakraborty and Martin [7] allowing discount pricing policies for inventory subject to declining demand, Wee [36] for economic production lot-size model for deteriorating items with partial backordering, Wee and Yu [37] for deteriorating inventory model with temporary price discount, Mandal and Maiti [24] for inventory models of breakable items with AUD, IQD and stock-dependent demand. In a discounted system of breakable items, a retailer faces a conflicting situation. A retailer is tempted to go for large purchase to avail the advantage of price discount, but he risks an increased number of damaged units due to large inventory. Hence the retailer tries to opt for an optimum decision balancing the above two factors. Till now, all the works on discounted inventory control systems are confined to a single item only. For multi-items problems with AUD having a resource constraint, the usual solution algorithm for the single item becomes useless, as possible solutions may exist in different price intervals for different items. For the optimum one, the solutions are tried satisfying the resource constraints, say space constraint for multi-items. Hence, it is almost impossible to apply the conventional price-break algorithm in the case of a multi-item inventory system having number of price-breaks on each item. But use of GA removes all these complexities and makes the decision-making problem much simpler. As, in GA, the decision variables, that is the quantities of different items are selected randomly, the respective prices are accordingly selected from the logic structure of the algorithm for price-breaks and the corresponding profit function is evaluated satisfying the required space constraint. Out of the evaluated profits for different chromosomes in different iterations, the maximum one is selected using genetic operators-selection, crossover and mutation.

In this paper, a multi-item inventory model is developed for breakable items with price discount. Here demand and damage functions are both stock-dependent. For this model, AUD, IQD and a combination of AUD and IQD in the form of two price breaks are allowed. The items are stored in a limited storage space. To use general form of shape and scale parameters of breakability function, the integrals are computed numerically by Simpson's 1/3rd rule. GA is developed in real code representation using Roulette wheel selection, arithmetic crossover and uniform mutation. This algorithm with some logic structures connecting the price-break values has been implemented to solve the above inventory models and the optimum order quantities are determined in order to have maximum possible profit satisfying the space constraint in equality sense. To illustrate the model along with the proposed algorithm, numerical examples are presented and sensitivity analyses are performed for some parameters of demand and damage functions.

2. GA algorithm

Genetic algorithm (GA) approach was first developed in 1975 by Holland [17] using the name 'genetic plan', and it attracted considerable attention as a methodology for search, optimization and learning after Goldberg's [15] publication. GAs are stochastic search methods for optimization problems based on the mechanics of natural selection and natural genetics that is the principle of evolution –'survival of the fittest'.

GA uses the following three genetic operators – reproduction, crossover and mutation. To solve a decision–making problem, the above genetic operations are performed sequentially and repeatedly.

The generalized GA procedure using the above components is given below.

begin
$t \to 0$
initialize Population (t)
evaluate Population (t)
while (not terminate condition)
{
$t \to t + 1$
select Population (t) from Population $(t - 1)$
alter (crossover and mutation) Population (t)
evaluate Population (t)
}
print optimum result
end

2.1. *Constraints handling in GA*

The main idea of handling constraints lies in (i) an elimination of equalities and (ii) careful design of special genetic operators which guarantee to keep all chromosomes within the feasible solution set. To ensure that the chromosomes are feasible, we have to check all the new chromosomes generated by genetic operators. We suggest that a function is designated for each target optimization problem, the output value 1 means that the chromosome is feasible, 0 infeasible. For example, we can make an objective function

$Z(Q) = \sum_{i=1}^{n} Z_i \ (Q_i\ )$ where $Q = (Q_1\ , Q_2\ , \ldots,\ Q_n)$ and $Q_1, Q_2, \ldots, Q_n$ are decision variables.
subject to $(g_j\ (Q_1\ , Q_2, \ldots, Q_n\ ) \ \leq\ b_j)$
as follows:

```
for j = 1 to m do
if (g_j (Q_1 , Q_2 , ..., Q_n ) ≤ b_j)
      continue;
   else
      return 0;
   endfor
      return 1
```

2.2. *Notations and assumptions*

The following notations are used in the proposed model:

n = number of items,
W = total storage area or volume available.

Parameters for the i-th item ($i = 1, 2, \ldots, n$)
$q_i(t)$ = inventory level at any time t (decision variable),
H_i = inventory carrying cost per item per unit time,
P_i = selling price,
T_i = time length for each cycle,
C_{3i} = set-up cost per cycle,
Q_i = maximum inventory level,
D_i = demand rate,
b_{i1}, b_{i2} = first and second price break points of the i-th item respectively,
$p_i(= p_{i1}, p_{i2}, p_{i3})$ = unit purchasing price,
m_{i1}, m_{i2} = discount rates ($0 < m_{i1}, m_{i2} < 1$) for the i-th item,
w_i = space required by one unit of i-th item,
Q_i^* = optimum value of Q_i,
$Z(Q_1, Q_2, \ldots Q_n)$ = profit function.

The inventory model is developed under the following assumptions.

(i) Shortages are not allowed.
(ii) The replenishment is infinite.
(iii) The lead time is zero.
(iv) The selling price is fixed on the basis of purchasing cost. Let s_i fraction on the purchasing cost be made as profit and then the selling price is fixed as

$$P_i = p_i(1 + s_i),\quad 0 \leq s_i < 1 \tag{1}$$

(v) Demand $D_i(q_i)$ depends directly on the current inventory level and is of the form

$$D_i(q_i) = \alpha_i + \beta_i q_i$$

where $\alpha_i, \beta_i > 0$ are called scale and shape parameters of the demand function.

(vi) Breakable units $B_i(q_i)$ is a known function of current inventory level and is of the form

$$B_i(q_i) = a_i q_i^{\gamma i}$$

where $\gamma_i(0 < \gamma_i < 1)$ and $a_i(0 < a_i < 0)$ are scale and shape parameters of the breakage function.

(vii) The unit carrying cost H_i are x_i percentage of the unit purchasing cost.

(viii) The purchasing price is offered under the following AUD scheme:

$$p_i = \begin{cases} \$p_{i1}, & 0 < q_i < b_{i1} \\ \$p_{i2}, & b_{i1} \le q_i < b_{i2} \\ \$p_{i3}, & q_i \ge b_{i2} \end{cases} \tag{2}$$

(ix) The purchasing price under IQD is:

The unit-purchasing price under IQD system is p_i for $0 < q_i < b_{i1}$, $p_i(1 - m_{i1})$ for additional quantity over b_{i1} but less than b_{i2} and $p_i(1 - m_{i2})$ for any additional quantity over b_{i2}. Thus the total purchase cost becomes

$p_i q_i$ for $q_i < b_{i1}$

$p_i b_{i1} + p_i(1 - m_{i1})(q_i - b_{i1})$ for $b_{i1} \le q_i < b_{i2}$ and

$p_i b_{i1} + p_i(1 - m_{i1})(b_{i2} - b_{i1}) + p_i(1 - m_{i2})(q_i - b_{i2})$ for $q_i \ge b_{i2}$

Therefore the general unit price becomes

$$\begin{array}{ll} p_{i1} = \$p_i, & 0 < q_i \le b_{i1} \\ p_{i2} = \$p_i(1 - m_{i1}) + p_i m_{i1} b_{i1}/q_i, & b_{i1} < q_i \le b_{i2} \\ p_{i3} = \$p_i(1 - m_{i2}) + p_i(m_{i2} - m_{i1}) b_{i2}/q_i + p_i m_{i1} b_{i1}/q_i, & q_i > b_{i2} \end{array} \tag{3}$$

For both systems, $p_{i1} > p_{i2} > p_{i3}$ for $i = 1, 2$.

3. Mathematical formulation

With the above assumptions, for the i-th item the differential equation which describes the variation of the inventory level $q_i(t)$ with respect to time t during the interval $[0, T_i]$ is

$$\frac{dq_i(t)}{\mathrm{d}t} = -D_i(q_i) - B_i(q_i), \quad 0 \le q_i < Q_i \tag{4}$$

with the boundary conditions,

$$q_i(t) = \begin{cases} Q_i, & \text{at } t = 0 \\ 0, & \text{at } t = T_i \end{cases}$$

The time length for each cycle is

$$T_i = \int_0^{T_i} \mathrm{d}t = \int_0^{Q_i} \frac{\mathrm{d}q_i}{D_i(q_i) + B_i(q_i)} \tag{5}$$

Total holding cost for each cycle is $H_i G_i(Q_i)$ where

$$G_i(Q_i) = \int_0^{T_i} q_i(t)\mathrm{d}t = \int_0^{Q_i} \frac{q_i dq_i}{D_i(q_i) + B_i(q_i)} \tag{6}$$

The total damaged units per cycle is

$$\theta_i(Q_i) = \int_0^{T_i} B_i(q_i)\mathrm{d}t = \int_0^{Q_i} \frac{B_i(q_i) dq_i}{D_i(q_i) + B_i(q_i)} \tag{7}$$

The net revenue per cycle is

$$N_i(Q_i) = s_i p_i(Q_i - \theta_i(Q_i)) - p_i\theta_i(Q_i) \tag{8}$$

Hence, the average profit during T_i is given by

$$Z(Q) = Z(Q_1, Q_2, \ldots, Q_n) = \sum_{i=1}^{n} Z_i(Q_i) = \sum_{i=1}^{n} \frac{[N_i(Q_i) - H_i G_i(Q_i) - C_{3i}]}{T_i} \tag{9}$$

$$\text{subject to} \sum_{i=1}^{n} w_i Q_i \leq W \tag{10}$$

Now, the problem is to maximize the average profit function $Z(Q_1, Q_2, \ldots, Q_n)$ and to find the optimum value of Q_i satisfying the constraint (10) and taking the price break (2) for AUD , price break (3) for IQD and price breaks (2) and (3) for combined AUD and IQD systems into account.

4. Working procedure of GA with AUD, IQD, AUD and IQD

As mentioned in article 2. GA is developed as follows:

(a) Parameters

The different parameters on which this GA is developed are population size (POPSIZE), probability of crossover (P_c) , probability of mutation (P_m) and maximum number of generation (MAXGEN). In this case, POPSIZE $= 100$, $P_c = 0.3$, $P_m = 0.1$ and MAXGEN $= 5000$.

(b) Representation

A n-dimensional real vector $Q = (q_1, q_2, \ldots, q_n)$ is used to represent a solution where q_i represents optimum inventory: (decision variables) level for i-th item ($i = 1, 2, \ldots, n$)

(c) Initialization

N such solutions $Q_1, Q_2, \ldots, Q_N$ are randomly generated by a random number generator such that each solution satisfies the resource constraints of the problem.

(d) Fitness value

To evaluate the value of the objective function $Z(Q)$ due to the potential solution $Q = (q_1, q_2, \ldots, q_n)$, purchase cost of q_i is taken as p_{i1} per unit if $0 < q_i < b_{i1}$, p_{i2} per unit if

$b_{i1} \leq q_i < b_{i2}$ and p_{i3} per unit if $q_i \geq b_{i2}$ for $(i = 1, 2, \ldots, n)$ (c.f. equations (2) and (3)). Following this, holding costs for the items are also calculated.

(e) Selection process to create a new population

The following steps are followed for this purpose

(i) Find total fitness of the population $F = \sum_{i=1}^{N} Z(Q_i)$
(ii) Calculate the probability of selection f_i of each solution Q_i by the formula $f_i = Z(Q_i)/F$.
(iii) Calculate the cumulative probability cp_i for each solution Q_i by the formula $cp_i = \sum_{j=1}^{i} f_j$
(iv) Generate a random number ‘r’ in the range [0,1].
(v) If $r < cp_1$ then select Q_1 otherwise select $Q_i (2 \leq i \leq N)$ where $\mathrm{cp}_{i-1} \leq \mathrm{r} < \mathrm{cp}_i$.
(vi) Repeat steps (iv) and (v) N times to select N solutions from the old population. Clearly one solution may be selected more than once.
(vii) Selected solution set is denoted by $P^1(T)$ in the proposed GA algorithm.

(f) Crossover

(i) Selection for crossover: For each solution of $P^1(T)$, generate a random number r from the range [0,1]. If r < P_c then the solution is taken for crossover, where P_c is the probability of crossover.
(ii) Crossover process: Crossover takes place on the selected solutions. For each pair of coupled solutions Y_1, Y_2 a random number c is generated from the range [0,1] and Y_1, Y_2 are replaced by their offspring’s Y_{11} and Y_{21} respectively where $Y_{11} = cY_1 + (1 - c)Y_2, Y_{21} = cY_2 + (1 - c)Y_1$, provided Y_{11}, Y_{21} satisfied the constraints of the problem. Let the new set of solution be denoted by $P^{11}(T)$

(g) Mutation

(i) Selection for mutation: For each solution of $P^{11}(T)$ generate a random number r from the range [0,1]. If r < P_m then the solution is taken for mutation, where P_m is the probability of mutation.
(ii) Mutation process: To mutate a solution $Q = (q_1, q_2, \ldots, q_n)$, select a random integer r in the range [1,n]. Then replace q_r by randomly generated value within the boundary of r^{th} component of Q.

5. Logic structure for unit price and holding cost

The algorithm for price discount in the case of inventory system with two breakable items and three price breaks is:

5.1. *AUD scheme*

If $(q_1 \geq b_{12})$
{
calculate revenue, holding cost and then average profit with $p_1 = p_{13}$
}

else if ($q_1 \geq b_{11}$ and $q_1 < b_{12}$)
{
calculate revenue, holding cost and the average profit with $p_1 = p_{12}$
}
else
{
calculate revenue, holding cost and the average profit with $p_1 = p_{11}$
}

The above price structure is repeated for the 2nd item, q_2

5.2. *IQD scheme*

For this system, the unit prices are given in (3) and after the calculation of the unit prices, the same logic structure as mentioned in AUD is followed.

5.3. *AUD and IQD scheme*

Obviously, here AUD logic structure is used for the 1st item, say q_1 and IQD structure for 2nd item, say q_2.

6. Numerical illustration

The following values of inventory parameters in Table 1 are used for AUD, IQD and combined AUD and IQD systems to calculate optimum value of profit function $Z(Q_i)$ along with optimum inventory level Q_i by GA and results are presented in Tables 2–4. Here, the inventory model is formulated for two items.

Table 1. Different parametric values for different systems: (input data).

Discount system	i	α_i	β_i	γ_i	C_{3i}($)	a_i	s_i	x_i(%)	m_{i1}(%)	m_{i2}(%)
AUD	1	45	0.25	0.75	50	0.3	0.3	4	–	–
	2	30	0.30	0.75	45	0.2	0.4	5	–	–
IQD	1	45	0.25	0.75	50	0.3	0.3	4	0.18	0.20
	2	30	0.30	0.75	45	0.2	0.4	5	0.20	0.10
AUD and IQD	1	45	0.25	0.75	50	0.3	0.3	4	–	–
	2	30	0.30	0.75	45	0.2	0.4	5	0.20	0.10

$w_1 = 2$ sq mt., $w_2 = 3$ sq. mt., $W = 590$ sq. mt.,

Table 2. Optimum values of ordered quantities, damaged units and profit for different discount systems.

Discount system	Q_1^*	Q_2^*	θ_1	θ_2	Z ($)
AUD	145.91	99.35	14.00	6.91	201.48
IQD	135.91	92.97	12.68	6.30	171.94
AUD and IQD	137.03	94.06	12.82	6.40	215.30

Table 3. Optimum profit when β_i varies in AUD, IQD, AUD and IQD systems.

(β_1, β_2)	Q_1^*	Q_2^*	θ_1	θ_2	T_i	Z ($)
AUD						
(0.3, 0.35)	140.60	96.78	12.62	6.32	2.03	220.30
(0.35, 0.4)	142.34	97.25	12.21	6.07	1.96	239.38
(0.40, 0.45)	145.00	99.35	11.95	5.96	1.92	258.73
(0.45, 0.50)	145.90	99.00	11.52	5.68	1.85	276.34
(0.50, 0.55)	144.03	99.70	10.86	5.49	1.79	293.46
IQD						
(0.3, 0.35)	126.28	86.59	10.88	5.44	1.88	186.92
(0.35, 0.4)	127.18	86.95	10.49	5.28	1.82	201.33
(0.40, 0.45)	129.58	89.05	10.29	5.16	1.78	215.83
(0.45, 0.50)	129.46	99.60	9.85	4.93	1.75	227.92
(0.50, 0.55)	129.01	88.64	9.42	4.73	1.66	242.93
AUD and IQD						
(0.3, 0.35)	141.06	96.79	12.67	6.33	2.02	234.94
(0.35, 0.4)	142.04	100.15	12.18	6.30	1.99	253.28
(0.40, 0.45)	142.04	100.15	11.63	6.02	1.92	272.20
(0.45, 0.50)	140.83	98.30	11.00	5.62	1.83	290.38
(0.50, 0.55)	139.10	95.45	10.39	5.20	1.74	307.76

Table 4. Optimum profit when a_i varies in AUD, IQD, AUD and IQD systems.

(β_1, β_2)	Q_1^*	Q_2^*	θ_1	θ_2	T_i	Z ($)
AUD						
(0.3, 0.35)	137.48	94.25	14.77	7.88	2.03	179.90
(0.35, 0.40)	134.59	93.05	16.11	9.12	1.99	158.60
(0.4, 0.45)	130.00	89.19	16.98	9.86	1.92	138.71
(0.45, 0.50)	130.54	92.35	18.68	11.61	1.90	117.44
(0.50, 0.55)	128.00	87.69	19.68	11.97	1.83	98.61
IQD						
(0.3, 0.35)	130.89	90.14	13.79	7.41	1.98	154.27
(0.35, 0.40)	132.70	91.62	15.79	8.93	1.96	135.98
(0.4, 0.45)	132.70	91.62	17.48	10.23	1.94	117.99
(0.45, 0.50)	133.00	89.53	19.17	11.14	1.93	99.14
(0.50, 0.55)	133.26	95.36	20.81	13.41	1.91	81.12
AUD and IQD						
(0.3, 0.35)	134.59	93.05	14.34	7.74	2.02	193.15
(0.35, 0.40)	130.00	90.25	15.35	8.75	1.95	172.17
(0.4, 0.45)	130.80	90.95	17.13	10.13	1.93	150.69
(0.45, 0.50)	130.27	90.95	18.62	11.38	1.90	129.58
(0.50, 0.55)	132.16	89.92	20.57	12.39	1.87	107.72

6.1. *AUD system*

Let, for the 1st item

$$c_{p1} = \begin{cases} \$12.00, & 0 < q_1 < 100 \\ \$11.25, & 100 \le q_1 < 200 \\ \$10.00, & q_1 \ge 200 \end{cases}$$

For the 2nd item

$$c_{p2} = \begin{cases} \$13.00, & 0 < q_2 < 50 \\ \$12.00, & 50 \le q_2 < 100 \\ \$11.00, & q_2 \ge 100 \end{cases}$$

For this price structure, the optimum values of the decision variables and the profit function are evaluated by GA maximizing (9) with (10) and presented in Table 2.

6.2. *IQD system*

Following equation (3), the following unit prices are assumed.
For the 1st item

$$\begin{aligned}
p_{11} &= \$10, && 0 < q_1 \le 50\\
p_{12} &= \$p_{11}(1-m_{11}) + p_{11}m_{11}b_{11}/q_1, && 50 < q_1 \le 100\\
p_{13} &= \$p_{11}(1-m_{12}) + p_{11}(m_{12}-m_{11})b_{12}/q_1 + p_{11}m_{11}b_{11}/q_1, && q_1 > 100
\end{aligned}$$

For the 2nd item:

$$\begin{aligned}
p_{21} &= \$13, && 0 < q_2 \le 40\\
p_{22} &= \$p_{21}(1-m_{21}) + p_{21}m_{21}b_{21}/q_2, && 40 < q_2 \le 100\\
p_{23} &= \$p_{21}(1-m_{22}) + p_{21}(m_{22}-m_{21})b_{22}/q_2 + p_{21}m_{21}b_{21}/q_2, && q_2 > 100
\end{aligned}$$

For this price structure, maximizing (9) with (10) , the optimum values of the decision variables and the maximum profit function are evaluated and presented in Table 2.

6.3. *AUD and IQD system*

Here, AUD scheme is allowed for the 1st item and IQD scheme for the 2nd item. Following the price structure of equation (2) for AUD scheme and equation (3) for IQD scheme, we have
For the 1st item

$$p_1 = \begin{cases} \$13.00, & 0 < q_1 < 40 \\ \$11.75, & 40 \le q_1 < 100 \\ \$10.75, & q_1 \ge 100 \end{cases}$$

For the 2nd item

$$\begin{aligned}
p_{21} &= \$15, && 0 < q_2 \le 50\\
p_{22} &= \$p_{21}(1-m_{21}) + p_{21}m_{21}b_{21}/q_2, && 50 < q_2 \le 100\\
p_{23} &= \$p_{21}(1-m_{22}) + p_{21}(m_{22}-m_{21})b_{22}/q_2 + p_{21}m_{21}b_{21}/q_2, && q_2 > 100
\end{aligned}$$

For this price structure, as before, the optimum values of the decision variables and the maximum profit are calculated and presented in Table 2.

7. Sensitivity analysis

For the above-mentioned general models, two types of sensitivity analyses are performed. We evaluate the effect of changes in the parameters 'a_i' (the coefficient of damaged units) and 'β_i' (the shape parameter of the demand function) on the average profit, taking these changes one at a

Table 5. Optimum profit when γ_i varies in AUD system.

(γ_1, γ_2)	Q_1^*	Q_2^*	θ_1	θ_2	Z ($)
(0.45, 0.45)	139.43	97.45	4.07	2.24	280.88
(0.50, 0.50)	139.43	97.45	4.91	2.70	273.67
(0.55, 0.55)	139.43	97.45	5.99	3.24	264.81
(0.60, 0.60)	139.43	97.45	7.32	3.89	253.64
(0.65, 0.65)	139.43	97.45	8.92	4.67	239.39
(0.70, 0.70)	145.23	98.98	11.46	5.73	222.20
(0.75, 0.75)	145.23	98.98	13.91	6.87	201.54

time and holding all other parameters at their optimal values. The results are presented graphically for AUD, IQD and AUD and IQD systems.

(a) Effect of changing the shape parameter (β_i) of demand function on total average profit:

Here, the assumed demand function is $\mathrm{D}_i = \alpha_i + \beta_i q_i$. Increasing β_i by 0.05 successively, the optimum order quantities and total average profit are calculated and presented in Table 3.

From Table 3, it is evident that the model is more sensitive to the shape parameters β_i . It is also observed that as β_i increase, profit increases. When β_i is increased by 20% for the AUD, IQD and (AUD and IQD) systems, the profit is increased approximately by 8.66%, 7.7% and 7.8% respectively. As β_i increase, obviously demand increases and to meet the increased demand, either initial stock level has to be increased or the time period for each replenishment is reduced. Here, it is observed that for increased demand, $T_i's$ have been successively decreased as there is only marginally change in Q_i^*'s. Moreover, there is no specific trend of behaviour in Q_i^*'s.

(b) Effect of changing the scale parameter (a_i) of breakability function on total average profit:

Here, in each case, as expected, the increase in the co-efficient of damage function, a_i decreases the profit. When a_i is changed by 20%, the profit goes down approximately by 11.84%, 11.85% and 10.86% for AUD, IQD and (AUD and IQD) systems respectively. For all systems, the amount of breakable items increases and this brings down the total profit. Here, the system by itself tries to minimize the loss due to increased breakability by reducing the time periods for each cycle. It is supported by the fact that for each discount system values of $T_i's$ decrease successively. As before, there is not much change and no specific trend in the values of Q_i^*'s.

(c) Effect of changing the shape parameter (γ_i) of breakability function on total average profit:

From Table 5, it is seen that profit decreases with the increase of shape parameter of breakage function. As the items are more damaged when γ_i increases. Similar phenomenons are observed for the case of IQD and combination of these two.

8. Conclusion

In this paper, for the first time, a realistic multi-item inventory model with AUD, IQD and combination of these two has been formulated with a resource constraint and successfully solved by GA, the stochastic optimization process. Due to the complexities, till now, none has attempted this type of multi-item problem with space constraint by conventional price-break methodology. Though the model has been illustrated with only three price breaks, the GA developed here can be easily extended to include more than the three price-break points. Some interesting results relevant to this research paper have been presented in different discount forms. This model can be extended to include the finite time horizon, variable demand, etc. This problem can also be formulated in fuzzy, probabilistic and mixed environments.

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References

[1] P.L. Abad, *Joint price and lot size determination when supplier offers incremental quantity discount*, J. Oper. Res. Soc. 39 (1998), pp. 603–607.
[2] P.L Abad, *Determining optimal selling price and lot size when the supplier offers all unit quantity discount*, Decis. Sci. 19(3) (1988), pp. 632–634.
[3] F. Altiparmak, M. Gen, L. Lin *et al.*, *A genetic algorithm approach for multi-objective optimization of supply chain networks*, Comput. Indus. Eng. 51(3) (2006), pp. 196–215.
[4] R.C. Baker and T.L Urban, *A deterministic inventory system with an inventory-level-dependent demand rate*, J. Oper. Res. Soc. 39 (1988), pp. 823–831.
[5] M. Ben-Daya and A. Raouf, *On the constrained multi-item single-period inventory problem*, Int. J. Prod. Manage. 13 (1993), pp. 104–112.
[6] E.C. Benton, *Multiple price breaks and alternative purchase lot-sizing procedures in material requirements planning systems*, Int. J. Prod. Res. 23 (1985), pp. 1025–1047.
[7] A.K. Chakravorty and G.E. Martin, *Discount pricing policies for inventories subject to declining demand*, Nav. Res. Logist. 36 (1989), pp. 489–502.
[8] C.W. Churchman, R.L. Ackoff, and E.L. Arnoff. *Introduction to Operations Research*, Wiley, New York, 1957.
[9] T.K. Datta and A.K. Pal, *Order level inventory system with power demand pattern for items with variable rate of deterioration*, Indian J. Pure Appl. Math. 19(1) (1988), pp. 1043–1053.
[10] T.K. Datta and A.K. Pal, *A note on inventory model with inventory-level-dependent demand rate*, J. Oper. Res. Soc. 41(10) (1990), pp. 971–975.
[11] L. Davis (Ed.). *Hand book of Genetic Algorithms*, New York, Van Nostrand Reinhold, 1991.
[12] S.M. Disney, M.M. Naim, and P.R. Towill. *Genetic algorithm optimization of a classical inventory control systems*, Int. J. Prod. Econ. 68 (2000), pp. 259–278.
[13] L. Gaafar, *Applying genetic algorithm to dynamic lot-sizing with batch ordering*, Comput. Ind. Eng. 51(3) (2006), pp. 433–444.
[14] E. Gelsema (Ed.) *Special issue of genetic algorithms*, Pattern Recognit. Lett. 16 (1995), p. 8.
[15] D.E.Goldberg. *Genetic Algorithm in Search, Optimization and Machine Learning*, Massachusetts, Addision Wesley, 1989.
[16] S.N. Guria, D. Maiti, and A.K. Bhunia, *On the multi-item EOQ models with two storage facilities*, Model. Measure. Control, 21 (2000), pp. 31–40.
[17] J.H. Holland. *Adaptive in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, 1975.
[18] S. Kar, T.K. Roy and M. Maiti, *Multi-item inventory model with probabilistic price dependent demand and imprecise goal and constraints*, Yugoslav J. Oper. Res. 11 (2001), pp. 93–104.
[19] M.K. Maiti and M. Maiti, *Production policy for damageable items with variable cost function in an imperfect production process via genetic algorithm*, Math. Comput Model. 42 (2005), pp. 977–990.
[20] A.K. Maiti and M. Maiti, *An application of real-coded genetic algorithm (RCGA) for mixed integer non-linear programming in two-storage multi-item inventory model with discount policy*, Appl. Math. Comput. 183 (2006), pp. 903–915.
[21] A.K. Maiti and M. Maiti, *Two-storage inventory model with random planning horizon*, Appl. Math. Comput. 183 (2006), pp. 1084–1097.
[22] M.K. Maiti and M. Maiti, *Fuzzy inventory model with two-warehouses under possibility constraints*, Fuzzy Sets Syst. 157 (2006a), pp. 52–73.
[23] M.K. Maiti and M. Maiti, *Two-storage inventory model with lot size dependent fuzzy lead-time under possibility constraints via genetic algorithm*, Eur. J. Oper. Res. Available online: www.sciencedirect.com, 2006b.
[24] M. Mandal and M. Maiti. *Inventory models for breakable items with quantity discount and stock-dependent demand*, Model. Measure. Control, 42 (1996), pp. 47–62.
[25] M. Mandal and M. Maiti. *Inventory of damagable items with variable replenishment rate, stock-dependenr demand and some units in hand*, Appl. Math. Model. 23 (1999), pp. 799–807.
[26] M. Mandal and M. Maiti. *Inventory of damageable items with variable replenishment and stock-dependent demand*, Asia Pacific J. Oper. Res. 17 (2000), pp. 41–54.
[27] B.N. Mandal and S. Phaujdar. *A note on inventory model with stock-dependent consumption rate*, OPSEARCH, 26 (1989), pp. 43–46.
[28] Z. Michalewicz. *Genetic Algorithms + Data Structures = Evolution Programs*, Berlin, Springer, 1992.
[29] S. Mondal and M. Maiti, *Multi-item fuzzy EOQ models using genetic algorithm*, Comput. Indus. Eng. 44 (2002), pp. 105–117.
[30] G. Padmanabhan and P. Vrat, *Analysis of multi-item inventory system under resource constraint: A non-linear goal programming approach*, Eng. Cost Prod. Manage. 13 (1990), pp. 104–112.

[31] S.K. Pal, S. De, and A. Ghosh. *Designing Hopfield type networks using genetic algorithms and its caparisons with simulated annealing*, Int. J. Patterns Recognit. Artif. Intell. 11 (1997), pp. 447–461.

[32] R. Sarkar and Charles Newton. A genetic algorithm for solving economic lot size scheduling problem, Comput. Indus. Eng. 42 (2002), pp. 189–198.

[33] E.A. Silver and R. Peterson, *Decision Systems for Inventory Management and Production Planning*, John Wiley, New York, 1985.

[34] J.C. Smith, *A genetic algorithm approach to solving a multiple inventory problem*, Int. J. Eng. Theory, 10(1) (2003), pp. 45–54.

[35] T.L Urban, *An inventory model with an inventory level dependent demand rate and relaxed terminal conditions*, J. Oper. Res. Soc. 43(7) (1992), pp. 721–724.

[36] H.M Wee, *Economic production lot-size model for deteriorating items with partial backordering*, Comput. Indust. Eng. 24 (1995), pp. 449–458.

[37] H.M. Wee and C.P. Yu, *A deterministic inventory model with a temporary price discount*, Int. J. Prod. Econ. 53 (1997), pp. 81–90.

[38] Y-W. Zhou, *Optimal replenishment problem for items with polynomial demand during their life period*, J. Syst. Eng. Electron. 10(2) (1999), pp. 9–14.

[39] Y-W. Zhou, *An optimal replenishment policy for items with inventory-level-dependent demand and fixed life time under the LIFO policy*, J. Oper. Res. Soc. 54(6) (2003), pp. 585–593.