6
Plurals and Second-Order Logic

We have encountered three ways to talk about the many, using primitive plurals, set theory, or mereology. Let us now examine a fourth and final way, namely using second-order logic. We begin with a brief introduction to second-order logic. We will then examine whether this system can be eliminated in favor of plural logic or vice versa.

6.1 Second-order logic

Consider the statement that Socrates thinks, which we formalize as:

(6.1) $T(s)$

Classical *first-order logic* allows us to generalize into the noun position occupied by 'Socrates' to conclude that there is an object x that thinks:

(6.2) $\exists x\, T(x)$

By allowing additional forms of generalization, we can obtain more expressive logics. *Second-order logic* (SOL) studies another form of generalization: it allows us to generalize into the predicate position occupied by T in (6.2) to conclude:

(6.3) $\exists F\, F(s)$

Following Frege, we describe the values of monadic second-order variables as *concepts*. So we gloss (6.3) as follows: there is a concept, F, such that Socrates falls under F.[1]

[1] Different glosses are found in literature, e.g. that a concept "applies to" an individual, that an individual is "in the extension of" a concept, or that an individual "instantiates" a concept. We will make use of these glosses when stylistically convenient.

The Many and the One: A Philosophical Study of Plural Logic. Salvatore Florio and Øystein Linnebo,
Oxford University Press.
DOI: 10.1093/oso/9780198791522.003.0006

Variables taking predicate position—called *second-order variables*—belong to special sorts and are written as upper-case letters. There is one sort for each type of predicate. First, we have a sort for variables taking the position of monadic predicates. Variables of this sort are marked by the superscript '1' ($X^1, Y^1, \ldots$). Then, we have another sort for variables taking the position of binary predicates. Variables of this sort are marked by the superscript '2' ($X^2, Y^2, \ldots$). And so on. When no confusion arises, we omit the superscripts.

Second-order logic is thus a multi-sorted system, with a sort for individual variables and multiple sorts for second-order variables. As mentioned in Section 2.6, monadic second-order logic (MSOL) is the subsystem of second-order logic that adds to first-order logic only monadic variables. We can expand MSOL with predicates taking monadic variables as argument. We refer to the resulting system as MSOL+.

The key observation in this context is that a monadic second-order term allows us to talk about many things simultaneously. For a concept can be used to represent all the things that fall under it. For example, the concept F represents precisely the φs if and only if $\forall x(Fx \leftrightarrow \varphi(x))$.

Monadic second-order logic must nevertheless be carefully distinguished from plural logic. While the former allows us to generalize into predicate position, the latter allows us to generalize plurally into noun position. Plural logic thus allows us to infer from (6.1) that there are one or more objects xx that think:

(6.4) $\exists xx \forall y\, (y \prec xx \rightarrow T(y))$

As is apparent, plural and monadic second-order logic permit different kinds of generalization.

This difference distinguishes MSOL from the three other approaches to representing many objects simultaneously that we considered above. While all four approaches enable us to talk about "collections" of some given objects, MSOL is unique among these approaches in representing the "collections" by means of the semantic values of predicates. So, on the second-order approach, there will be interactions between ordinary first-order predication and the representation of "collections" that are not found in any of the other approaches.

Let us describe the second-order logic that we will adopt. The rules associated with the singular vocabulary—logical connectives and quantifiers—

are the usual ones, for example introduction and elimination rules for each logical expression. Second-order quantifiers (that is, the quantifiers binding second-order variables) have introduction and elimination rules analogous to those of the singular quantifiers. In addition, there is the second-order comprehension scheme:

(SO-Comp) $$\exists F \forall x(Fx \leftrightarrow \varphi(x))$$

where F does not occur free in φ, as well as the polyadic analogues of this scheme.[2]

Is there a natural language counterpart of second-order quantification? In other words, does quantification into predicate position occur in natural language? Some authors have defended an affirmative answer. For example, Higginbotham (1998, 3) points to the following sentence.[3]

(6.5) John is everything we wanted him to be.

A natural regimentation of (6.5) involves bound variables in predicate positions. If this analysis is correct, MSOL is not only an available language but it is actually in use.[4]

Even if there isn't always a good natural language counterpart, we need not give up on second-order languages. For the lack of correspondence might be due to an expressive limitation of natural language. Still, if we are to use a second-order language in theorizing, we need to learn it. Is it possible for us to do that? Williamson suggests that we use "the direct method":

> We may have to learn second-order languages by the direct method, not by translating them into a language with which we are already familiar. After all, that may well be how we come to understand other symbols in contemporary logic, such as ⊃ and ◇: we can approximate them by 'if' and 'possibly', but for familiar reasons they may fall short of perfect synonymy [. . .]. At some point we learn to understand the symbols directly; why not

[2] For ease of readability, we will often omit parentheses around argument positions that immediately follow second-order variables.

[3] See also Rayo and Yablo 2001.

[4] A fan of plurals might attempt a further generalization as well, namely to pluralize not only first-order variables but also higher-order ones. For example, one might try to regiment "You are several things that I am not", using a plural version of second-order logic, as $\exists FF\ \forall G(G \prec FF \rightarrow G(\text{you}) \wedge \neg G(\text{I}))$. See Fine 1977 for a system with a very general form of pluralization.

use the same method for $\forall F$? We must learn to use second-order languages as our home language. (Williamson 2003, 459)

If successful, this method establishes the legitimacy of second-order resources.

6.2 Plural logic and second-order logic compared

What is the relation between second-order and plural logic? Let us compare the languages of MSOL+ and PFO+. We can suppose that the singular first-order fragments of these languages coincide. Each language adds to this shared base a new stock of variables, as well as new predicates whose argument places admit such variables. Although the new variables of each language function in very different ways, there is (as we will now explain) a natural correspondence between their values.

The new variables of MSOL+ are supposed to range over monadic concepts, which (as Frege pointed out) we may think of as functions from individuals to truth values.[5] For example, one of these variables can replace the predicative expression '... is a boy' and have as its value the function β defined as follows:

$$\beta(x) = \begin{cases} \text{the true,} & \text{if } x \text{ is a boy;} \\ \text{the false,} & \text{if not.} \end{cases}$$

By contrast, each of the new variables of PFO+ is allowed to have one or more individuals as its values. For example, one of these variables can replace the plural noun phrase 'the boys' and have as its many values all and only the boys in the domain, say *bb*.

We can now explain the promised natural correspondence between the values of the new kinds of variable. The correspondence is nicely illustrated by the function β and the plurality *bb*. Suppose we start with β. Then *bb* can be defined as all and only the objects that β maps to the true. Suppose instead we start with the plurality *bb*. Then β can be defined as the function that maps these objects, and only these, to the true. As mathematicians like to put it, β is *the characteristic function* of *bb*. This allows us to define either β or *bb* in terms of the other.

[5] Although our target is a syntactic translation between MSOL+ and PFO+, it is convenient to refer to semantic concepts such as *values* and *ranges* of variables. This is done for ease of exposition. We turn to semantic matters in Chapter 7.

Although this single example captures the essence of the comparison we want to make between MSOL+ and PFO+, it is useful to state things in proper generality. This requires some notation for talking about the various types of expression. As customary, let e be the type of ordinary singular terms and t be the type of truth values. Moreover, for any two types θ_1 and θ_2, we let $\langle \theta_1, \theta_2 \rangle$ be the type of functions from entities of type θ_1 to entities of type θ_2. Thus, $\langle e, t \rangle$ is the type of one-place predicates, which (as we have seen) stand for functions from individuals to truth values. But we make a single important addition to this customary setup: we add another basic type, *ee*, as the type of plural terms.

Using this notation, we can rehearse the above explanation—only now stated in proper generality. While MSOL+ adds variables of type $\langle e, t \rangle$, PFO+ adds variables of type *ee*. But there is a natural correspondence between the two types *ee* and $\langle e, t \rangle$. Given any objects *aa*, there is an associated function α that sends an object x to the true just in case $x \prec aa$. We may think of α as the semantic value of the predicative expression '... is one of *aa*'. Conversely, given any function α of type $\langle e, t \rangle$, there is—at least according to traditional plural logic—a plurality *aa* of all and only those objects that α sends to the true. Finally, each of MSOL+ and PFO+ adds predicates applying to any number of arguments of type e and of the one additional type available in that language, namely $\langle e, t \rangle$ or *ee*.

With these explanations on board, it is easy to describe translations between the two languages. The basic idea is simply to map each variable xx_i to X_i and vice versa, and to map a predicate with an argument place of type *ee* to a predicate with a corresponding argument place of type $\langle e, t \rangle$ and vice versa. For example, 'cooperate(xx)' is mapped to 'cooperate(X)'.

There is only one small bump in the road. While a monadic concept may apply to no individuals at all, a plurality is ordinarily taken to consist of at least one individual. This bump is easily handled by incorporating into the translations a trick due to Boolos, which we described in Appendix 4.A. The trick is nicely illustrated by considering the behavior of the resulting translations on existentially quantified statements. Let σ be the translation from the plural to the second-order language. Then a plural existential generalization is translated as the corresponding conceptual existential generalization restricted to non-empty concepts, that is:

$$\exists xx\, \varphi \overset{\sigma}{\longmapsto} \exists X(\exists y Xy \wedge \sigma(\varphi))$$

Let τ be the translation in the opposite direction. Then a conceptual existential generalization is translated as a disjunction:

$$\exists X\,\varphi \quad \overset{\tau}{\longmapsto} \quad \exists xx\,\tau(\varphi) \vee \tau(\varphi')$$

where φ', defined as the result of substituting '$x_i \neq x_i$' everywhere for '$X_j x_i$', is a way of simulating existential generalizations involving empty concepts.

Equipped with these translations, let us now consider the axioms of the two theories. It can be shown that the comprehension axioms of the two theories match via the translations. A second-order comprehension axiom

$$\exists X \forall x(Xx \leftrightarrow \varphi(x))$$

translates as a formula equivalent to the corresponding plural comprehension axiom:

$$\exists x\varphi(x) \rightarrow \exists xx\forall x(x \prec xx \leftrightarrow \varphi(x))$$

And clearly, a plural comprehension axiom translates as a formula equivalent to a second-order comprehension axiom.

It remains only to consider some additional axioms of plural logic. The axiom stating that every plurality is non-empty translates as the trivial claim that every non-empty concept is non-empty. More interestingly, a plural indiscernibility axiom is translated as a corresponding indiscernibility principle in the second-order language:

$$\forall x(Xx \leftrightarrow Yx) \rightarrow (\varphi(X) \leftrightarrow \varphi(Y))$$

Indiscernibility principles of this kind are implausible on a conceptual interpretation of the second-order language, and for that reason are not part of MSOL+, as defined above. Two coextensive concepts might be discerned by modal properties. Assume, for example, that *being a creature with a heart* and *being a creature with a kidney* are coextensive. Even so, these two concepts can be discerned by a modal property such as *possibly being instantiated by something that lacks a heart*. We return shortly to the philosophical significance of this observation.

6.3 The elimination of pluralities in favor of concepts

The formal results just presented open up the possibility of two eliminative strategies. We might try to eliminate pluralities in favor of concepts, or we might try to effect the opposite elimination. Michael Dummett advocates the former option:

> [A] plural noun-phrase, even when preceded by the definite article, cannot be functioning analogously to a singular term. [...] [I]t is only as referring to a concept that a plural phrase can be understood [...]. To say that it refers to a concept is to say that, under a correct analysis, the phrase is seen to figure predicatively. (Dummett 1991, 93)

As an illustration of his eliminative strategy, he proposes to analyze (6.6) as (6.7) and (6.8) as (6.9).[6]

(6.6) All whales are mammals.

(6.7) If anything is a whale, it is a mammal.

(6.8) The Kaiser's carriage is drawn by four horses.

(6.9) There are four objects each of which is a horse that draws the Kaiser's carriage.

Here, plurals nouns ('whales', 'mammals', 'horses') are replaced by corresponding singular predicates ('is a whale', 'is a mammal', 'is a horse'); moreover, the plural 'four objects' is eliminated in the usual way in favor of first-order quantifiers and identity statements.

To see how Dummett's proposal could be generalized, it is useful to start from a basic example of collective predication:

(6.10) Russell and Whitehead wrote *Principia Mathematica*.

How should this use of plurals be eliminated? One option is to use Dummett's analysis of (6.8) as a model and to regiment (6.10) as (6.11):

(6.11) Russell is a co-writer of *Principia Mathematica*, Whitehead is as well, and no one else is.

However, the formal translation given in Section 6.2 suggests a more systematic approach. First, we regiment (6.10) in PFO+. Then, we apply the formal translation to eliminate pluralities in favor of concepts. The steps are as follows.

[6] For discussion, see for example Rumfitt 2005 and Oliver and Smiley 2016, Chapter 4.

(6.10) Russell and Whitehead wrote *Principia Mathematica*.

(6.12) $\exists xx(\forall y(y \prec xx \leftrightarrow (y = r \vee y = w)) \wedge \text{wrote}(xx, p))$

(6.13) $\exists X(\forall y(Xy \leftrightarrow (y = r \vee y = w)) \wedge \text{wrote}(X, p))$

We refer to this eliminative approach as the *predicative analysis*.

This analysis has received much criticism.[7] Before going into details, a general observation is in order. A precondition for eliminating plurals in favor of second-order resources is that plural logic can be interpreted in second-order logic.[8] The objections we will now consider purport to show that the former theory cannot even be interpreted in the latter.

One objection concerns *flexible predicates*, which can combine felicitously with both singular and plural terms. We observed in Section 2.3 that 'own a house' and 'lifted a boat' appear to be flexible. However, the translation defined in Section 6.2 implicitly assumes that there are no flexible predicates. Thus, if such predicates are added to the language, the interpretability result from that section is no longer available.[9]

Oliver and Smiley (2016, 59) discuss the problem, calling attention to the following sentence:

(6.14) Wittgenstein wrote the *Tractatus*, not Russell and Whitehead.

The predicate 'wrote' appears to be flexible, and a natural formalization of this sentence in plural logic would employ a single predicate applying to both Wittgenstein and the duo Russell and Whitehead:

(6.15) $\text{wrote}(w_1, t) \wedge \exists xx(\forall y(y \prec xx \leftrightarrow (y = r \vee y = w_2)) \wedge \neg\text{wrote}(xx, t))$

Applying the predicative analysis to (6.15) yields:

(6.16) $\text{wrote}(w_1, t) \wedge \exists X(\forall y(Xy \leftrightarrow (y = r \vee y = w_2)) \wedge \neg\text{wrote}(X, t))$

[7] See especially Yi 1999, Yi 2005, and Oliver and Smiley 2016, Chapter 4.

[8] At least, all theoretically useful parts of our plural logic must be so interpretable. If some aspects of this theory were found to be of no scientific use, they might perhaps be abandoned and therefore ignored for the purposes of the elimination.

[9] The need for flexible predicates disappears if one adopts a one-sorted plural logic. In this system, individuals become singleton pluralities. By raising the type of individuals to that of pluralities, one restores uniformity among the argument places of predicates.

However, this sentence is not even well-formed in MSOL+. According to the standard formulation of second-order logic, predicates are *strictly typed*: each of their argument places can be occupied by expressions belonging to a unique sort. So the first argument of 'wrote' cannot be an individual variable in one conjunct and a second-order variable in the other.

In response, the proponent of the predicative analysis could relax the requirement that predicates be strictly typed and allow certain predicates to apply to objects and concepts alike. In the context of higher-order logic, this flexibility is known as *cumulativity*. It is not assumed in standard presentations of second-order logic but there is no formal obstacle to adopting it. A cumulative version of higher-order logic is perfectly consistent. Indeed, Gödel once referred to strict typing as a "superfluous restriction" of higher-order logic (Gödel 1933, 46). In sum, if we allow flexible predicates in plural logic, we can allow flexible predicates in second-order logic too. If second-order logic is modified in this way, we can at least *simulate* predicates of the former kind using predicates of the latter kind.

In linguistics, a variant of the predicative analysis has been proposed by Higginbotham and Schein (1989). It centers around an event-based account of predication and resembles closely the combination of events and mereology discussed in Section 5.6. There we mentioned two ways of analyzing plural predications in terms of events. According to the first, a mereological sum can serve as the agent of the event described by the predicate. According to the second, a mereological sum can serve to represent the atoms that function as co-agents of the event, that is, participate in the event as agents. It is easy to see that the role played by mereological sums in each account could be played by concepts. A concept can serve as the agent of an event or, perhaps more plausibly, it can serve to represent the individuals (namely its instances) that function as co-agents of the event.

Higginbotham and Schein (1989) develop the second approach and analyze (6.17) as (6.18):

(6.17) Some apostles lifted the piano.

(6.18) $\exists X\big(\exists y Xy \wedge \forall y(Xy \rightarrow \text{apostle}(y)) \wedge \exists e(\text{lift-the-piano}(e) \wedge \forall y(Xy \leftrightarrow y \text{ is a co-agent of } e))\big)$

This approach avoids the objection from flexible predicates. To see why, let us return to the example that illustrated the need for such predicates:

(6.14) Wittgenstein wrote the *Tractatus*, not Russell and Whitehead.

On Higginbotham and Schein's analysis, the predicate 'wrote' applies to events, not to objects or concepts. This means that the predicate's arguments are uniform. Applying their analysis to (6.14) yields someting along the following lines:

$$\begin{aligned}(6.19)\quad & \exists e(\text{writing-the-}\mathit{Tractatus}(e) \land w_1 \text{ is an agent of } e) \land \\ & \neg\exists e\exists X\big(\text{writing-the-}\mathit{Tractatus}(e) \land \forall y(Xy \leftrightarrow (y = r \lor y = w_2)) \land \\ & \qquad \forall y(Xy \leftrightarrow y \text{ is a co-agent of } e)\big)\end{aligned}$$

So the objection from flexible predicates does not get off the ground.

A second objection to the predicative analysis concerns *extensionality*. The thought is that, because pluralities and concepts differ with respect to extensionality, the former cannot be eliminated in favor of the latter. A clear manifestation of this problem has already emerged. At the end of Section 6.2, we remarked that the main difficulty for interpreting PFO+ in MSOL+ has to do with the indiscernibility principle, which states that coextensive pluralities satisfy the same formulas. The principle strikes most logicians and philosophers as highly plausible and is included in our axiomatization of plural logic. Its second-order translation states that coextensive concepts satisfy the same formulas:

$$\forall x(Xx \leftrightarrow Yx) \rightarrow (\varphi(X) \leftrightarrow \varphi(Y))$$

However, this second-order indiscernibility principle is not plausible (see the example on p. 109), let alone a good candidate for a logical truth. Still, the principle is required if our translation is to yield an interpretation of PFO+ in MSOL+.

Let us illustrate this objection with an example discussed by Yi (1999, 2005). Consider the following inference, which is intuitively valid:

(6.20)

Russell and Whitehead cooperate.
Russell and Whitehead are philosophers who wrote *Principia Mathematica*.

Some philosophers who wrote *Principia Mathematica* cooperate.

The inference can be formalized in PFO+ as:

$$\text{(6.21)}\quad \frac{\begin{array}{l}\exists xx(\forall y(y \prec xx \leftrightarrow (y = r \vee y = w)) \wedge \text{cooperate}(xx)) \\ \exists xx(\forall y(y \prec xx \leftrightarrow (y = r \vee y = w)) \wedge \\ \qquad\qquad \forall y(y \prec xx \rightarrow \text{philosopher}(y)) \wedge \text{wrote}(xx, p))\end{array}}{\exists xx(\forall y(y \prec xx \rightarrow \text{philosopher}(y)) \wedge \text{wrote}(xx, p) \wedge \text{cooperate}(xx))}$$

We obtain a formalization of the inference in MSOL+ by applying to (6.21) the translation procedure described in Section 6.2. The result is as follows:

$$\text{(6.22)}\quad \frac{\begin{array}{l}\exists X(\forall y(Xy \leftrightarrow (y = r \vee y = w)) \wedge \text{cooperate}(X)) \\ \exists X(\forall y(Xy \leftrightarrow (y = r \vee y = w)) \wedge \\ \qquad\qquad \forall y(Xy \rightarrow \text{philosopher}(y)) \wedge \text{wrote}(X, p))\end{array}}{\exists X(\forall y(Xy \rightarrow \text{philosopher}(y)) \wedge \text{wrote}(X, p) \wedge \text{cooperate}(X))}$$

As is easy to verify, the conclusion of each formal argument can be derived from its premises with the help of instances of the appropriate indiscernibility principle. Without them, the validity of the initial inference would be left unexplained.

Since (6.20) is logically valid and does not appear to be enthymematic, the indiscernibility principles must be assumed to be logical. In particular, it must be assumed that *as a matter of logic* a predicate like 'cooperate' is extensional in the sense that it does not distinguish between coextensive concepts. Yi concludes:

> it is one thing to hold the extensional conception, quite another to hold, more implausibly, that the truth of the conception rests on logic alone. [. . .] One cannot meet the objections [. . .] under the assumption that the property indicated by "*COOPERATE*" is one that Russell calls *extensional* (that is, a second-order property instantiated by any first-order property coextensive with one that instantiates it). This does not help unless the assumption holds by logic [. . .]. (Yi 2005, 475; see also Yi 1999, 173)

Is this objection fatal to the eliminative project under consideration? We think not. Let a *first-level concept* be a concept of objects, and a *second-level concept* be a concept of first-level concepts.[10] Then the key observation is that

[10] Thus, a first-level concept can be the value of a second-order variable, and more generally, a concept of level n can be the value of a variable of order $n + 1$.

the following fact is provable in a basic extension of MSOL+: every second-level concept has a counterpart that doesn't discern between coextensive first-level concepts. That is, for every second-level concept **P**, there is another second-level concept $\mathbf{P}^*$ applying to all and only the first-level concepts coextensive with those to which **P** applies:

$$\mathbf{P}^*(X) \leftrightarrow \exists Y(\mathbf{P}(Y) \wedge \forall x(Xx \leftrightarrow Yx))$$

Let us call $\mathbf{P}^*$ *the undiscerning counterpart of* **P**. The proponent of the eliminative strategy can use these undiscerning counterparts to capture the extensional behavior of plural predicates.[11] This move does not require that the second-order indiscernibility principle be logical and is indeed consistent with some failures of the principle. In sum, we can admit undiscerning second-level concepts alongside "discerning" ones. We just need to ensure that all plural predicates are translated by means of undiscerning second-level concepts. This shows that plural logic can be *simulated* using concepts.

In fact, there is another reason to think that the objection from extensionality is not fatal. We have seen that the problem posed by flexible predicates can be avoided if we combine plurals and events along the lines indicated by Higginbotham and Schein. Remarkably, their framework also manages to avoid the objection from extensionality. Consider again the potentially problematic inference (6.20):

(6.20)

Russell and Whitehead cooperate.
Russell and Whitehead are philosophers who wrote *Principia Mathematica*.

Some philosophers who wrote *Principia Mathematica* cooperate.

Its validity can easily be explained in Higginbotham and Schein's framework. Roughly put, the premises are understood as stating that Russell and Whitehead are co-agents of events of three kinds: cooperating, being a philosopher, and writing *Principia Mathematica*. It follows from second-order comprehension that there is a concept X whose instances are co-agents of events of those three kinds. But, on Higginbotham and Schein's analysis, this is precisely what the conclusion states.

[11] See Florio 2014a, 12.

A third and final objection is a modal analogue of the objection from extensionality. Membership in a plurality is typically taken to be modally rigid. That is, if *a* is one of *bb*, then necessarily so, at least on the assumption that the objects in question exist; and likewise with non-membership. (See Chapter 10 for a defense of this view.) By contrast, falling under a concept is almost universally taken to be modally non-rigid. Although Socrates in fact falls under the concept *philosopher*, he might not have done so. Thus, when modalities are added, we obtain an extension of PFO+ that may no longer be interpreted in the corresponding extension of MSOL+.

One might try to counter this objection by imitating the response given to the extensionality problem. We saw that it is possible to model the behavior of plural predicates using undiscerning second-level concepts, even though many such concepts are discerning. Something similar might work here. It might be possible to model the rigid behavior of pluralities using rigid concepts, even though many such concepts are non-rigid. The crucial question, though, is what assurance we have that the requisite rigid concepts exist. In the presence of plural logic, a compelling argument for their existence is available. For every plurality *aa*, we can use second-order comprehension to define a corresponding concept:

$$\exists F \Box \forall x (Fx \leftrightarrow x \prec aa)$$

The rigidity of the plurality ensures the rigidity of the concept defined in terms of it. In the absence of plural logic, however, it is unclear that the existence of rigid concepts can be motivated without invoking highly controversial forms of modalized second-order comprehension (see Williamson 2013, Chapter 6). We conclude that the most compelling reason against the elimination of pluralities in favor of concepts has to do with their modal behavior. The modal rigidity of pluralities isn't easily secured in second-order logic without relying on plurals.

6.4 The elimination of concepts in favor of pluralities

Let us now consider the attempt to eliminate in the opposite direction, that is, to eliminate concepts in favor of pluralities. As we will see, this project is more challenging and therefore more likely to fail.[12]

[12] See Williamson 2003, Section IX, with whom we are in broad agreement.

Some difficulties have to do with the fact that there are some natural ways to generalize MSOL+ that have no obvious analogues in the case of plurals. One example is third-order quantification, that is, quantification over second-level concepts. This raises the question of whether there is any plural analogue of quantification of third and higher orders. A monadic second-level concept would correspond to a plurality of pluralities. We are thus led to the question of superplurals, which has surfaced in our discussion from time to time. We defer a proper discussion of the matter to Chapter 9.

Another example concerns polyadic concepts. There are not only monadic concepts but also polyadic ones. If it is permissible to quantify over monadic concepts, it should be equally permissible to quantify over polyadic ones. By contrast, there is no obvious polyadic analogue of plural quantification.[13] So it is unclear whether polyadic second-order logic can be interpreted in some form of plural logic.

One might respond that the desired polyadic analogues can be defined provided that ordered pairs are available. Suppose that for all individuals a and b, there is an ordered pair $\langle a, b\rangle$ such that:

$$\langle a, b\rangle = \langle a', b'\rangle \leftrightarrow a = a' \land b = b'$$

Using ordered pairs, it is easy to define a plural analogue of any relation. For example, a plurality of ordered pairs can be used to represent the extension of a dyadic relation, provided that the relation is non-empty. This plural analogue of a relation might perhaps be criticized for being too indirect and unnatural. Just as a monadic concept of ordered pairs can represent a dyadic relation but isn't *really* one, a plurality of ordered pairs too can represent a dyadic relation but isn't *really* one.

Although we find talk about what relations "really are" somewhat nebulous, a clear objection can be extracted from the fog. Consider again the mismatch between the modal profile of pluralities and that of concepts. We observed in the previous section that it is possible to model the behavior of pluralities using rigid concepts, even though concepts are in general non-rigid. But the reverse direction is problematic. With only modally rigid material at our disposal we are unable to model any non-rigid phenomenon. For example, while a plurality of ordered pairs can model *the extension* of a dyadic relation, it cannot in general represent all of its *intensional* features.

[13] See Hewitt 2012a for an attempt to develop a non-obvious analogue.

Yet another reason against the elimination of concepts in favor of pluralities emerges in the final chapter, where we argue that the comprehension scheme found in traditional plural logic must be restricted. In particular, we deny that there is a plurality of absolutely every object, which renders our system unable to represent the universal concept.

6.5 Conclusion

We have compared plural and second-order logic. As part of this undertaking, we have investigated to what extent one of these systems can be interpreted in the other.

We found that second-order logic in its entirety cannot be interpreted in plural logic. It is unclear how to handle polyadic concepts or concepts of higher levels, and, most seriously, there is no way to handle the intensionality of second-order logic, using only modally rigid pluralities. Since interpretability is a precondition for elimination, we conclude that second-order logic cannot be eliminated in favor of plural logic.

What about the other direction? We found that plural logic can be interpreted in terms of second-order logic, at least in the absence of modality. This raises the question of whether plural logic should be eliminated in favor of a subsystem of second-order logic. We regard the proposed elimination as problematic, for several reasons. The identification is *prima facie* implausible because of the deep and pervasive differences in how plurals and predication are represented in English and other natural languages. Being a member of a plurality and falling under a concept are, it seems, simply different things.

Our detailed analysis identifies a further, more robust, reason against the proposed elimination. As already stressed, a precondition for this elimination is the interpretability of plural logic in second-order logic. We have isolated various assumptions that are needed for this interpretability result to obtain. First, we found that plural predicates are extensional, while predicates of concepts are not. We showed how to circumvent this problem by invoking undiscerning second-level concepts. Second, we observed that plural membership and predication have different modal profiles. While the former is a matter of necessity (at least conditional on the continued existence of all the objects in question), the latter is not. This difference was neutralized, in our interpretability result, only by the assumptions that plural terms stand for rigid concepts. But the most natural and compelling argument for the existence of such concepts seems to rely on plurals.

The conclusion of this chapter echoes those of the preceding ones. There are several ways to represent many objects simultaneously, at least in ordinary circumstances where our domain is a set. In addition to taking plurals at face value, we may use sets, mereological sums in the individual sense, or monadic concepts. Although these systems share a common formal structure, at least in ordinary circumstances, the notions that they represent are different and must be kept apart.

What if ordinary circumstances do not obtain? In Chapter 12, we provide an account of domains that do not form a set and argue that, in such domains, another deep difference between plurals and predicates emerges: while the former are subject to a form of limitation of size, the latter are not. If correct, this account provides yet another reason not to attempt to eliminate second-order logic in favor of plural logic.

Let us briefly summarize all of Part II. We have examined four different ways to talk about many objects simultaneously. By identifying various philosophical and formal differences, we have argued that none of these ways should be eliminated in favor of any other. Thus, the detailed, pairwise comparisons of Part II yield an argument for primitive plurals that Chapter 3 failed to produce.