7
The Semantics of Plurals

Plural logic has emerged as an appealing framework for the regimentation of natural language plurals and for the development of various philosophical projects. As we will now see, however, the choice of a regimenting language leaves wide open the semantic question of how such a language should be interpreted. Since plural logic is characterized by a precise axiomatic theory, one may wonder why we should be interested in its semantics.

The semantic question is important, for at least two reasons. First, a semantics is needed for a complete account of plural logic. It is by means of a semantics that we define a relation of logical consequence, which can be used to identify valid as well as invalid arguments. The axioms of plural logic help us reason correctly but, by themselves, do not tell us which arguments are invalid. This limitation can be overcome by studying not only which meanings the expressions of the language have but also which meanings such expressions *might* have. The part of semantics concerned with possible meanings is model theory. In model theory, logical consequence is defined as truth preservation under every interpretation (model) of the language, where an interpretation is simply an assignment of possible meanings. Starting from the notion of interpretation, we thus obtain a fully general way of characterizing whether or not a conclusion is a logical consequence of some premises. Second, the alleged features of plural logic that underlie the philosophical applications discussed in Section 2.6—ontological innocence, expressive power, and absolute generality—are really semantic features and hence can only be assessed in light of a worked-out semantics.

In this chapter, we are primarily concerned with traditional plural logic. In particular, the various semantic accounts we consider validate the unrestricted axiom scheme of plural comprehension. These accounts have to be adjusted if, as we suggest in Chapter 12, an alternative logic is adopted.

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7.1 Regimentation vs. semantics

The regimentation of plurals was discussed in Chapter 3, where we focused on the following two views.

> Regimentation Singularism
>
> A singular language suffices to regiment a fragment of natural language containing plurals, where a language is said to be "singular" if it has no plural resources, unlike, say, $\mathcal{L}_{\text{PFO}+}$.
>
> Regimentation Pluralism
>
> Plural terms, variables, and predicates are required to regiment the relevant fragment of natural language.

We asked whether singularism can provide a satisfactory regimentation of plurals and found that, though often benign, this approach has some shortcomings as a general strategy. Especially if we assume that absolute generality is possible and that traditional plural logic is valid, there are good reasons to favor regimentation pluralism. In light of this, we examined the relation between plural logic and other systems.

While regimentation is relevant to semantics, it does not determine how semantic interpretations should be specified, at least not in any obvious way. So it is important to distinguish two questions that are often conflated:

(Q1) How should a given fragment of natural language be regimented?
(Q2) Once a regimenting language has been chosen, how should we specify the semantic interpretations of that language?

While the first question is entirely about the object language, the second is also about the metalanguage.

The importance of the distinction between regimentation and semantics becomes clear when we look at other cases of semantic analysis. Consider, for example, modalities. The first question concerns the proper regimentation of modal notions. Should they be regimented as predicates of sentences? Or should they rather be regimented as operators? As is well known, considerations related to paradox strongly recommend the second approach. Once a particular regimentation has been chosen, a second question arises as to how semantic interpretations for the regimented language should be specified. The most popular option, embodied in standard possible world semantics, is simply to characterize models as set-theoretic constructions.

Modalities are therefore absent from the semantics. An alternative approach would take modal notions as primitive in the metalanguage and use them to define the interpretations of the regimented language. But, as the set-theoretic approach shows, the metalanguage need not embrace the notions being analyzed. This means that the semantics cannot be directly read off the regimented language. The transition from regimentation to semantics is a delicate one.

The case of plurals is no different: here too we have options. In perfect analogy with the case of modalities, we could provide a set-theoretic specification of the interpretations of the language, or we could take plural talk as primitive in the metalanguage and use it to formulate the semantics. For our purposes, it is useful to characterize two broad methodological approaches to the semantic question.

SEMANTIC SINGULARISM

Once the regimenting language has been chosen, semantic interpretations can be specified within a theory formulated in a singular metalanguage.

SEMANTIC PLURALISM

Once the regimenting language has been chosen, semantic interpretations must be specified within a theory formulated in a plural metalanguage.

The second approach was Boolos's great innovation and marks a clean break with broadly Quinean approaches, which might acknowledge the availability of plural resources but would not permit their use in rigorous theorizing.

Combining the two semantic approaches with the approaches to regimentation discussed earlier, we end up with four alternatives, shown in the table below together with some of their supporters.

	regimantation singularism	regimentation pluralism
semantic singularism	Quine	(some linguists)
semantic pluralism	—	McKay, Oliver & Smiley, Simons, Rayo, Yi

The bottom left-hand quadrant is empty because regimentation singularism makes semantic singularism almost inevitable. If plurals are just singular expressions in disguise, why should we appeal to plurals in the semantics? But as emphasized above, regimentation pluralism does not make semantic pluralism inevitable. If plurals are *not* just singular expressions in disguise, however, why avoid plurals in the semantics? Possible reasons have to do not only with simplicity and ideological economy, but also with expressibility problems that arise when one embraces type-theoretic hierarchies, as we will see in Chapter 11.

7.2 Set-based model theory

Model theory provides the standard setting for the characterization of logical properties. Given an object language $\mathcal{L}$, one first defines the notions of *interpretation* (or *model*) of $\mathcal{L}$ and *truth in an interpretation* (or *satisfaction*). Then one uses these notions to characterize the key relation of logical consequence for sentences of $\mathcal{L}$. A sentence ψ is said to be a logical consequence of a set of sentences Σ just in case ψ is true in every interpretation in which every member φ of Σ is true. When this holds, we write:

$$\Sigma \vDash \psi$$

Other logical properties (such as logical truth and consistency) can easily be defined in terms of consequence.

The possible interpretations of $\mathcal{L}$ are obtained by varying two features: the domain of quantification and the interpretation of the non-logical terminology of $\mathcal{L}$ (that is, its constants and predicates). Thus, an interpretation is fixed by specifying these two features. The second feature is specified by means of an *interpretation function* (or *interpretation*, for short).

The ordinary implementation of model theory is based on sets. Working within set theory, an interpretation of $\mathcal{L}$ is taken to be a pair $\langle d, f \rangle$, where d is a set representing the domain and f is an interpretation function from the non-logical terminology of $\mathcal{L}$ to set-theoretic constructions generated by d. When $\mathcal{L}$ is the language of first-order logic, the situation is familiar. For example, f assigns an element of the domain to singular constant of $\mathcal{L}$, and it assigns a subset of the domain to each monadic predicate.[1]

The next step is to define what it is for a sentence to be true in an interpretation. Again, the situation is familiar when $\mathcal{L}$ is the language of first-order logic. We obtain the definition of truth in an interpretation from the more general relation of truth in an interpretation relative to a

[1] Our semantics treats all terms as denoting, as is usually done. The semantics could, if desired, be generalized to allow non-denoting terms. For discussion of non-denoting terms in the context of plurals, see Oliver and Smiley 2016, Chapter 5.

variable assignment. Let $i = \langle d, f \rangle$ be an interpretation, and let s be a *variable assignment* relative to d, namely a function assigning an element of d to each variable of $\mathcal{L}$. We use the notation $[\![E]\!]_{i,s}$ as follows:

$$[\![E]\!]_{i,s} = \begin{cases} f(E), & \text{if } E \text{ is a constant or a predicate;} \\ s(E), & \text{if } E \text{ is a variable.} \end{cases}$$

(We may omit one or both subscripts when the intended notation is clear from context.) So $[\![E]\!]_{i,s}$ stands for the semantic value of E (that is, its "denotation") according to i or s.

To define when a formula φ is true in i relative to s, written $i \vDash \varphi\ [s]$, we proceed by induction on the complexity of φ via satisfaction clauses. If φ is an atomic formula, say St, then:

(Sat-A) $\quad i \vDash St\ [s]$ if and only if $[\![t]\!]_{i,s} \in [\![S]\!]_{i,s}$

Since we treat the identity predicate as logical, it is always interpreted homophonically (namely by means of the analogous predicate in the metalanguage). That is:

(Sat-=) $\quad i \vDash t_1 = t_2\ [s]$ if and only if $[\![t_1]\!]_{i,s} = [\![t_2]\!]_{i,s}$

If φ is a negation $(\neg\psi)$ or a conjunction $(\psi_1 \wedge \psi_2)$, then we have the obvious clauses:

(Sat-¬) $\quad i \vDash \neg\psi\ [s]$ if and only if it is not the case that $i \vDash \psi\ [s]$

(Sat-∧) $\quad i \vDash \psi_1 \wedge \psi_2\ [s]$ if and only if $i \vDash \psi_1\ [s]$ and $i \vDash \psi_2\ [s]$

If φ is an existential generalization $(\exists v\, \psi)$, then

(Sat-∃) $\quad i \vDash \exists v\, \psi\ [s]$ if and only if $i \vDash \psi\ [s(v/x)]$ for some $x \in d$

where $s(v/x)$ is an assignment just like s, with the possible exception that $s(v/x)$ assigns x to v.

We are now ready to define our target notion, namely truth in an interpretation. The definitions just given ensure that if φ is a sentence (that is, it has no free variable), we can ignore variable assignments. More precisely, φ is true in i relative to a variable assignment if and only if φ is true in i relative to any other variable assignment. Thus, we can define φ to be true in i, written $i \vDash \varphi$, if φ is true in i for some (equivalently, every) variable assignment.

So far, we have only recapitulated the standard, set-based definition of truth in an interpretation for the language of first-order logic. But there is a straightforward extension of this definition to the richer language $\mathcal{L}_{\text{PFO+}}$. An interpretation of this language is a pair $\langle d, f \rangle$, just as before, only that f now also assigns set-theoretic semantic values to plural constants and predicates. For example, f assigns a non-empty subset of d to every plural constant, and it assigns a (possibly empty) set of non-empty subsets of d to every monadic plural predicate. Likewise, a variable assignment s relative to d is extended by assigning a non-empty subset of d to each plural variable.

The extended definition of truth in an interpretation relative to an assignment is achieved by adding the following satisfaction clauses to the previous ones. If φ is an atomic plural predication, say Ptt, then

(Sat-PA) $$i \vDash Ptt\ [s] \text{ if and only if } [\![tt]\!]_{i,s} \in [\![P]\!]_{i,s}$$

For the special case of plural membership, we have:

(Sat-≺) $$i \vDash t \prec tt\ [s] \text{ if and only if } [\![t]\!]_{i,s} \in [\![tt]\!]_{i,s}$$

This means that the interpretation of plural membership does not vary: it always corresponds to set-theoretic membership. Finally, if φ is a plural existential $(\exists vv\, \psi)$, then

(Sat-P∃) $$i \vDash \exists vv\, \psi\ [s] \text{ if and only if } i \vDash \psi\ [s(vv/x)] \text{ for some non-empty } x \subseteq d$$

where $s(vv/x)$ is an assignment just like s, with the possible exception that $s(vv/x)$ assigns x to vv.

It is worth highlighting an important implication of the last clause: plural quantifiers are taken to range over the full powerset of the first-order domain d, minus the empty set.[2] This semantic treatment of plural quantifiers corresponds to a *standard* interpretation of second-order logic, that is, an

[2] Since the range of plural quantifiers is always determined by the first-order domain, there is no need to specify it separately.

interpretation in which the second-order quantifiers range over all subsets of the first-order domain. In the next chapter, we develop an analogue of Henkin semantics, which permits plural quantifiers to have a narrower range.

To explain the standard semantics with which we are currently concerned, it might help to consider a particular interpretation of $\mathcal{L}_{\text{PFO}}$. This example will also be useful later in our discussion. To keep things simple, let us assume that $\mathcal{L}_{\text{PFO}}$ contains only the following items:

A. *singular terms*: two constants t and r, plus the usual variables $(v, v_1, v_2, \ldots)$;
B. *plural terms*: two constants tt and rr, plus the usual variables $(vv, vv_1, vv_2, \ldots)$;
C. *singular predicates*: a monadic predicate S;
D. parentheses and the usual logical symbols ($\neg, \wedge, \exists, \prec$, etc.).

The interpretation we want to consider is $i = \langle d, f \rangle$, with $d = \{a, b, c\}$ and f defined by the following identities:

$$f(t) = a \qquad f(r) = b$$
$$f(tt) = \{a, b\} \qquad f(rr) = \{b, c\}$$
$$f(S) = \{a, b\}$$

Then, for example, the next two sentences are true in i:

(7.1) $r \prec tt$

(7.2) $\exists v(v \prec rr \wedge \neg Sv)$

That is easy to verify using the clauses given above. For (7.1), we reason like this:

$i \models r \prec tt$	if and only if
$i \models r \prec tt\ [s]$, for some s	if and only if
$[\![r]\!]_{i,s} \in [\![tt]\!]_{i,s}$, for some s	if and only if
$f(r) \in f(tt)$	if and only if
$b \in \{a, b\}$	

For (7.2), an analogous series of steps yields:

$i \vDash \exists v(v \prec rr \wedge \neg Sv)$ if and only if
$x \in \{b, c\}$ and $x \notin \{a, b\}$, for some $x \in d$

Similarly, we could verify that these two sentences are false in *i*:

(7.3) $t \prec rr$

(7.4) $\neg\exists v(v \prec rr \wedge Sv)$

It might also help to consider a particular interpretation of $\mathcal{L}_{\text{PFO+}}$. This example too will be useful later in our discussion. For simplicity, we assume that $\mathcal{L}_{\text{PFO+}}$ augments $\mathcal{L}_{\text{PFO}}$ with just one (monadic) plural predicate *P*. Our interpretation of PFO+ is an extension of $i = \langle d, f \rangle$, the interpretation of $\mathcal{L}_{\text{PFO}}$ presented just above. We only need to specify a semantic value for *P*. Let f^+ be an extension of *f* such that $f^+(P) = \{\{a, b\}\}$. Then $i^+ = \langle d, f^+ \rangle$ is an interpretation of $\mathcal{L}_{\text{PFO+}}$. It follows by construction that any sentence of $\mathcal{L}_{\text{PFO}}$ that is true in *i* remains true in i^+. Here are some sentences available in $\mathcal{L}_{\text{PFO+}}$ but not in $\mathcal{L}_{\text{PFO}}$:

(7.5) Ptt

(7.6) $\exists vv \neg Pvv$

(7.7) $\exists v \exists vv((Sv \wedge v \prec vv) \wedge Pvv)$

Using the appropriate clauses, it would be easy to verify that, in i^+, the first two sentences are true while the last one is false.

On the semantics just given, PFO and PFO+ have metalogical properties that distinguish them from first-order logic. Neither system is complete or compact, and both lack the Löwenheim-Skolem property. Indeed, both systems are able to provide categorical characterizations of arithmetic and analysis, and a quasi-categorical characterization of set theory. In this sense, the expressive power of both systems goes beyond that of first-order logic.

7.3 Plurality-based model theory

We have seen that the familiar set-based model theory is easily extended to PFO and PFO+. However, there is nothing inherent in the idea of a model theory that requires it to be set-theoretic. So why not adopt plural resources in the metalanguage and exploit these richer resources to represent the semantic values of plural expressions? This alternative approach, which we call *plurality-based model theory*, was initiated by Boolos (1985a). On Boolos's new semantic paradigm, the semantic value of a plural variable is not a set (or any kind of set-like object) whose members are drawn from the ordinary, first-order domain. Rather, a plural variable has many values from this ordinary domain and thus ranges plurally over it. This semantic approach to plurals has become very popular among philosophers.[3]

To develop a plurality-based model theory, we proceed much as before. Working within plural logic, we first define a notion of interpretation. Then, we use this notion to characterize, via satisfaction clauses, that of truth in an interpretation. And as before, we rely on variable assignments as an intermediate step.

Let us spell out these steps. In set-based model theory, we defined domains and interpretations functions as special kinds of set-theoretic objects. This allowed us to define an interpretation as a pair $\langle d, f \rangle$. These definitions were possible because the semantic values of terms and predicates were themselves objects. But now we want the semantic value of a plural term to be one or more objects. So an interpretation function can no longer be a function in the usual set-theoretic sense. We need a different strategy.

The model-theoretic characterization of logical consequence requires that we can quantify over interpretations: an argument is valid just in case it is truth-preserving under *every interpretation* of the language. Since our metalanguage has just two sorts of quantifiers—singular and plural—interpretations must be either objects or pluralities. Given the semantic shift sanctioned by Boolos's approach, it is natural to consider the idea that interpretations themselves might be pluralities rather than objects. As it turns out, this idea leads to a nice formulation of the plurality-based model theory.

If we postulate a pairing operation on objects, there is a relatively simple way to proceed. Recall that an interpretation is fixed by specifying a domain of quantification and the interpretation of the non-logical terminology of the language. We can represent a domain of quantification by pairing a conventional symbol, say the symbol '∃', with each element of the domain. If we want the domain to consist of the objects a and b, for example, we

[3] See, e.g., Oliver and Smiley 2005; Yi 2005; Yi 2006; McKay 2006, Chapter 3; Rayo 2006; and Oliver and Smiley 2016, Sections 11.5, 12.5, and 13.2.

will represent that by means of the pairs $\langle$'$\exists$', $a\rangle$ and $\langle$'$\exists$', $b\rangle$. (For simplicity, we omit the quotation marks in this type of ordered pairs and write: $\langle \exists, a\rangle$ and $\langle \exists, b\rangle$.) Similarly, we can represent an interpretation function by pairing the relevant expressions with their semantic value or values. For example, if we want to assign the plurality a and b to the term tt, we will do that by means of the pairs $\langle tt, a\rangle$ and $\langle tt, b\rangle$. An interpretation will just be a plurality ii of pairs representing the relevant semantic information. Among ii there will be pairs representing information about the domain and pairs representing an interpretation function. Quantifying over interpretations amounts to quantifying over the appropriate pluralities of pairs.

Let us illustrate the new definition of interpretation by showing how to convert the set-based interpretation $i = \langle d, f\rangle$ from the previous section into a plurality-based interpretation. This way of coding a plurality-based interpretation goes back to Boolos 1985a. First, the domain $d = \{a, b, c\}$ is represented by these three pairs:

$$\langle \exists, a\rangle \quad \langle \exists, b\rangle \quad \langle \exists, c\rangle$$

Call these pairs *dd*. Next, there is the interpretation function *f*, which was defined by the following identities:

$$\begin{array}{ll} f(t) = a & f(r) = b \\ f(tt) = \{a, b\} & f(rr) = \{b, c\} \\ f(S) = \{a, b\} & \end{array}$$

We can represent *f* by means of eight pairs:

$$\begin{array}{ll} \langle t, a\rangle & \langle r, b\rangle \\ \langle tt, a\rangle \, \langle tt, b\rangle & \langle rr, b\rangle \, \langle rr, c\rangle \\ \langle S, a\rangle \, \langle S, b\rangle & \end{array}$$

Call these pairs *ff*. The plurality-based interpretation corresponding to i is the plurality ii combining *dd* and *ff*. Thus ii consists of the eleven pairs shown above.

Our next goal is to provide the satisfaction clauses defining the relation of truth in a plurality-based interpretation relative to a variable assignment. In this context, a variable assignment is a plurality of pairs *ss* representing the assignment of an object to each singular variable and of one or more objects to each plural variable. For instance, an assignment *ss* containing precisely the pairs $\langle v, a\rangle$, $\langle vv, b\rangle$ $\langle vv, c\rangle$ is one that assigns *a* to *v* and the plurality *b* and *c* to *vv*. The notation for semantic values will follow our earlier convention. That is, we let the symbol $[\![E]\!]_{ii,ss}$ indicate the interpretation of *E* according to *ii* if *E* is a term or a predicate, and we let it indicate the assignment to *E* according to *ss* if *E* is a variable. In both cases, the result can be one or more things. For instance, if we consider the interpretation *ii* and the assignment *ss* just introduced, we have that $[\![tt]\!]_{ii,ss}$ indicates *a* and *b*, whereas $[\![vv]\!]_{ii,ss}$ indicates *b* and *c*.

We are finally ready to state the clauses that define when a formula φ is true in *ii* relative to *ss*, written $ii \vDash \varphi\ [ss]$. If φ is an atomic formula, say *St*, then:

(Sat-A*) $\quad ii \vDash St\ [ss]$ if and only if $[\![t]\!]_{ii,ss} \prec [\![S]\!]_{ii,ss}$

A small wrinkle needs to be ironed out. A predicate may obviously have an empty extension, but there is no empty plurality. This mismatch is easily handled, for example by always adding an arbitrary triple to the interpretation of any predicate. This convention will henceforth be implicit in model theories where predicates are given a plural interpretation.

For plural membership, we have:

(Sat-$\prec$*) $\quad ii \vDash t \prec tt\ [ss]$ if and only if $[\![t]\!]_{ii,ss} \prec [\![tt]\!]_{ii,ss}$

Notice that plural membership is always interpreted homophonically, in accordance with our decision to treat it as logical. If φ is a negation $(\neg\psi)$ or a conjunction $(\psi_1 \wedge \psi_2)$, then we have:

(Sat-$\neg$*) $\quad ii \vDash \neg\psi\ [ss]$ if and only if it is not the case that $ii \vDash \psi\ [ss]$

(Sat-$\wedge$*) $\quad ii \vDash \psi_1 \wedge \psi_2\ [ss]$ if and only if $ii \vDash \psi_1\ [ss]$ and $ii \vDash \psi_2\ [ss]$

Let *dd* be the domain of *ii*. If φ is a singular existential $(\exists v\, \psi)$, then

(Sat-$\exists$*) $\quad ii \vDash \exists v\, \psi\ [ss]$ if and only if $ii \vDash \psi\ [ss(v/x)]$ for some $x \prec dd$

where $ss(v/x)$ is an assignment just like *ss*, with the possible exception that $ss(v/x)$ assigns *x* to *v*. If φ is a plural existential $(\exists vv\, \psi)$, then

(Sat-P$\exists$*) $\quad ii \vDash \exists vv\, \psi\ [ss]$ if and only if $ii \vDash \psi\ [ss(vv/xx)]$ for some $xx \preccurlyeq dd$

where $ss(vv/xx)$ is an assignment just like ss, with the possible exception that $ss(vv/x)$ assigns xx to vv.

Plural quantification receives, again, a *standard* interpretation. Plural quantifiers are taken to range over *every* subplurality of the first-order domain. Interestingly, it is possible to formulate an alternative, Henkin-style semantics even within a plurality-based model theory. We develop this idea in the next chapter, where we also explore its significant philosophical implications.

We have obtained a definition of the relation of truth in an interpretation relative to a variable assignment for formulas of $\mathcal{L}_{\text{PFO}}$. However, our definition is carried out in a richer metalanguage, namely $\mathcal{L}_{\text{PFO+}}$. This is because the relation being defined ("φ is true in ii relative to ss") has a singular argument for formulas and two plural arguments, one for interpretations and one for assignments. So, in this setting, '... is true in ... relative to ...' is a plural predicate, which takes us beyond PFO into PFO+. This is not an accident but a manifestation of Tarski's theorem on the undefinability of truth. We examine this phenomenon more closely in Chapter 11. A consequence of immediate concern is that we should expect the model theory for PFO+ to require an even richer metalanguage (see Section 7.5).

As in the case of set-based model theory, the satisfaction of a sentence is independent of the choice of variable assignment. So we can define truth in an interpretation as follows. For any sentence φ, φ is true in ii, written $ii \vDash \varphi$, if φ is true in ii for some (equivalently, every) variable assignment.

Going back to the interpretation ii we used as an example, it is easy to verify that ii makes true the same sentences that were made true by its set-based counterpart i. Recall that

(7.1) $r \prec tt$

was shown to be true in i (p. 129). We can verify that this sentence is true in ii by means the following reasoning:

$ii \vDash r \prec tt$	if and only if
$ii \vDash r \prec tt\,[ss]$, for some ss	if and only if
$[\![r]\!]_{ii,ss} \prec [\![tt]\!]_{ii,ss}$, for some ss	if and only if
$b \prec a$ and b	

For another example, apply the above clauses to (7.2), which is easily seen to yield:

$ii \vDash \exists v(v \prec rr \wedge \neg Sv)$ if and only if

$x \prec b$ and c, and $x \nprec a$ and b, for some $x \prec dd$

where *dd* is the domain of *ii*. These truth conditions now have a pleasing homophonic feel.

We mentioned above some of the key metalogical properties of PFO on the set-based model theory. PFO turns out to have the same metalogical properties on the plurality-based model theory just outlined. In particular, PFO is neither complete nor compact, and it lacks the Löwenheim-Skolem property. So PFO continues to have more expressive power than first-order logic.

While the two model theories are on a par with regard to such metalogical properties, it has nevertheless been argued that there are reasons to prefer the plurality-based model theory over its set-based analogue. Let us turn to this issue.

7.4 Criticisms of the set-based model theory

There is an apparent element of artificiality in the set-based model theory. Plural terms are taken to denote sets. So plural quantification is interpreted as quantification over sets. By contrast, the plurality-based model theory does justice to the intuitive idea that a plural term does not stand for a set of things, but it stands for the things themselves. Intuitively, the term 'Paris and Rome' does not stand for the set of the two cities; it stands for the cities themselves. The plurality-based approach captures this intuitive idea. It assumes that a plural term refers *plurally* to some things, without the mediation of a set that stands proxy for them.

The issue becomes especially pressing when the things intuitively denoted by a plural term are too many to form a set. Consider a plural constant intended to refer to all the sets. Assuming traditional plural logic, we can construct a plurality-based interpretation in which this term refers plurally, as intended, to all sets.[4] There is no corresponding interpretation in the set-based model theory. We must interpret a plural term by means of a single set.

[4] If the correct plural logic is the "critical" one we propose in the final chapter, then this interpretation is unavailable. We discuss the semantic significance of this approach in Section 12.8.

But there is no set of all sets in standard set theory. So the semantic value of our constant cannot encompass all sets.

Another manifestation of this issue concerns the domain of quantification. By requiring that the domain of quantification be a set, the set-based model theory rules out any interpretation whose domain is too big to form a set. In particular, there is no set-based interpretation whose domain includes all sets. This means that there is no interpretation corresponding to the intended model of set theory. The set-based model theory is thus unable to capture all the intuitive interpretations of the language.

The plurality-based model theory avoids these limitations, again assuming traditional plural logic. Since the domain of quantification is given by a plurality, it is possible to represent a domain encompassing all sets. Consider all and only the pairs such that their first coordinate is the symbol '∃' and the second coordinate is a set. Plural comprehension and the existence of a pairing operation on objects jointly entail that there is a plurality of exactly those pairs. This plurality represents a domain encompassing all sets. Likewise, there is a plurality that represents a domain encompassing every object whatsoever. Therefore, the plurality-based model theory can be said to capture not only the intended interpretation of set theory but also absolute generality.

As stated, these considerations pertain to *intuitive* limitations of the set-based model theory, namely its inability to represent intuitive semantic values or intended interpretations. But are such considerations relevant to logic? We can move beyond the intuitive level by focusing on a key fact that has been implicit in our discussion. While every set-based interpretation can be converted into a plurality-based one, it is a consequence of the plural version of Cantor's theorem that the reverse claim isn't true.[5] This fact is relevant to logic. For logical consequence is defined by quantifying over every interpretation and hence depends on which interpretations are admitted. Let us elaborate on this claim.

Imagine two parties A and B wishing to characterize the relation of logical consequence for sentences of a given language $\mathcal{L}$. Suppose B has a richer conception of interpretation than A. That is, every interpretation

[5] The theorem states that the subpluralities of *xx* are strictly more numerous than the members of *xx*, provided that *xx* has two or more members (see Section 3.5). Using traditional plural logic, we let *xx* be the universal plurality—that is, the plurality of absolutely all objects. It follows that the pluralities are more numerous than the objects. Since any plurality can be the domain of a plurality-based interpretation, it follows in turn that the plurality-based interpretations are more numerous than the set-based interpretations, which are objects.

countenanced by *A* is also countenanced by *B*, but not the other way around. So, letting $\vDash_A$ and $\vDash_B$ be the two parties' consequence relations, we have:

$$\Delta \vDash_B \varphi \Rightarrow \Delta \vDash_A \varphi$$

for any set of sentences Δ and any sentence φ. But there is no guarantee that the opposite implication holds and hence no guarantee that the two relations of consequence are coextensive. The proponent of the set-based model theory and the proponent of the plurality based model theory are in the same situation as *A* and *B*.

When the language is first-order, Georg Kreisel's famous "squeezing argument" can be used to establish that the two model theories yield an equivalent relation of consequence (Kreisel 1967). Let $\vDash_P$ and $\vDash_S$ be the relation of consequence sanctioned, respectively, by the plurality-based model theory and by the set-based model theory. But let us restrict attention to first-order sentences. The argument goes as follows. In the preceding paragraph, we established that:

$$\Delta \vDash_P \varphi \Rightarrow \Delta \vDash_S \varphi$$

By the completeness theorem for first-order logic, we have

$$\Delta \vDash_S \varphi \Rightarrow \Delta \vdash \varphi$$

where $\vdash$ is the usual provability relation for first-order logic. Finally, we observe that the plurality-based account of consequence is sound with respect to this relation:

$$\Delta \vdash \varphi \Rightarrow \Delta \vDash_P \varphi$$

This closes the circle of implications. It follows that:

$$\Delta \vDash_S \varphi \Leftrightarrow \Delta \vDash_P \varphi$$

In sum, despite the fact that the the proponent of the plurality-based model theory has a richer conception of interpretation than its set-based competitor, their definitions of logical consequence yield exactly the same verdict for arguments involving first-order sentences.

An essential premise of Kreisel's argument is that the set-based relation of consequence satisfies the completeness theorem. But this premise might not hold when we move beyond first-order logic. In fact, it fails for PFO, which is not complete according to the plurality-based model theory presented above. So Kreisel's argument is not available for PFO. We can, however, get the same effect by appealing to set-theoretic reflection principles. For a simple example, consider the principle which asserts that any sentence of PFO that is true in the universe of sets is also true in some set-based model:

$$\text{(PR)} \qquad \varphi \rightarrow \exists\alpha\,(\varphi^{V_\alpha})$$

where φ^{V_α} is the result of restricting the quantifiers of φ to the set V_α, whose elements are all the sets of rank less than α (see Section 4.6). The principle (PR) turns out to be equivalent to the claim that any sentence that has a plurality-based model also has a set-based model (Shapiro 1987). This ensures the extensional equivalence of two definitions of *logical truth*: one in terms of truth in every set-based model, the other in terms of truth in every plurality-based model. An analogous result is available for the notion of *logical consequence*, although the required reflection principle is stronger than the one just mentioned (again, see Shapiro 1987).

These results may assuage the worries with which the section started. The inability of a model theory to represent some intuitive semantic values or intended interpretations need not have an effect on the logic. In particular, the apparent artificiality of the set-based model theory need not manifest itself at the level of logical consequence. For example, the model theory does not validate incorrect existential consequences of the kind discussed in Section 3.3. In other words, although plural terms have sets as semantic values, a sentences like *Ptt* does not logically entail that sets exist. A parallel case is that of predication in the usual set-based model theory for first-order logic. Predicates have sets as semantic values. Yet a predication like *St* does not logically entail that sets exists. Boolos (1984b, 448–9) insisted that "it doesn't follow just from the fact that there are some Cheerios in the bowl that, as some who theorize about the semantics of plurals would have it, there is also a set of them all." A set-based model theory does not sanction that it follows *logically* from the fact that there are some Cheerios in the bowl that there is also a set of them.

As far as logic is concerned, and assuming the appropriate reflection principle, we have found no reason to think that adopting a set-based model theory for PFO is any more problematic than adopting a set-based model theory for first-order logic. As we will see shortly, however, other considerations may help us decide which is the more appropriate type of model theory.

7.5 The semantics of plural predication

The interpretation of plural predicates raises a number of interesting questions. We presented a plurality-based model theory for PFO in Section 7.3. Let us now examine how this model-theoretic approach can be extended to PFO+. There are two main ways to proceed, depending on whether we want to give plural predicates an extensional or an intensional interpretation.

As formulated above, the plurality-based model theory for PFO incorporates an extensional treatment of singular predication. For the semantic value of a monadic singular predicate *S* is the plurality of objects to which *S* applies, that is, its extension. This choice of semantic value aligns with the choice of semantic value in the set-based model theory, where *S* is assigned the set of objects in the domain to which *S* applies.

One might instead take an *intensional* approach to predication and interpret predicates not as pluralities, but as properties. Suppose—if only temporarily—that properties are objects, and let us interpret *S* by means of the property σ. Then we could simply include the pair $\langle \exists, \sigma \rangle$ in our interpretation function. Recall the plurality-based interpretation function *ff* for $\mathcal{L}_{\text{PFO}}$ described in Section 7.3 (p. 132):

$$\begin{array}{ll} \langle t, a \rangle & \langle r, b \rangle \\ \langle tt, a \rangle \; \langle tt, b \rangle & \langle rr, c \rangle \; \langle rr, d \rangle \\ \langle S, a \rangle \; \langle S, b \rangle & \end{array}$$

On an intensional approach, *ff* would be replaced by the following plurality of pairs:

$$\begin{array}{ll} \langle t, a \rangle & \langle r, b \rangle \\ \langle tt, a \rangle \; \langle tt, b \rangle & \langle rr, c \rangle \; \langle rr, d \rangle \\ \langle S, \sigma \rangle & \end{array}$$

The clause for singular predication would be revised accordingly:

$$ii \vDash St\,[ss] \text{ if and only if } [\![t]\!]_{ii,ss} \text{ has } [\![S]\!]_{ii,ss}$$

That is, the truth of a singular predication *St* amounts to the fact that the semantic value of the term *t* (an object) has the semantic value of *S* (a property).

We thus have two approaches to singular predication, one extensional and one intensional. How do these approaches extend to *plural* predication?

Let us start with the extensional approach. Suppose the semantic value of a singular predicate S is a plurality of objects. Then the most natural choice of semantic value for a plural predicate P would be a "plurality of pluralities", or a "superplurality" as we have called it. The intuitive reason is this. The predicate S applies to objects and hence its semantic value is the plurality of the objects to which it applies. Since P applies to pluralities, its semantic value must be the plurality of pluralities to which it applies.

But what is a "plurality of pluralities"? Throughout the book, we use 'plurality' as a shorthand for a plural construction. Thus, to talk about "a plurality of dogs" is just a shorthand for talking about one or more dogs. It is controversial whether it makes sense to talk about "a plurality of pluralities" and hence whether the expressive resources needed to formulate an extensional version of plurality-based model theory for PFO+ are legitimate. We address the question of superplurals in Chapter 9. For the time being, we would like to make two remarks. First, it is relatively straightforward to develop a formal system of superplural quantification suitable to develop our model theory (Rayo 2006). Moreover, natural language offers at least some help. We can think of a superplurality as some things articulated into distinct subpluralities, such as: Russell and Whitehead, and Hilbert and Bernays, or: these things, those things, and these other things.[6]

Assuming the legitimacy of the expressive resources needed, we must find a way to incorporate the proposed interpretation of plural predicates into the model theory. We would like to proceed much as in the case of singular predicates, where a singular predicate S was interpreted by means of a plurality of ordered pairs, $\langle S, a\rangle$, $\langle S, b\rangle$, et cetera. Each of these ordered pairs, say $\langle S, x\rangle$, represents that S applies to x. Now consider a *plural* predicate P. We would like to interpret P by means of a bunch of ordered pairs, which we may think of as $\langle P, aa\rangle$, $\langle P, bb\rangle$, et cetera. Each of these ordered pairs, say $\langle P, xx\rangle$, represents that P applies to xx. The problem, however, is that no sense has yet been assigned to expressions such as '$\langle P, aa\rangle$'. After all, an ordered pair is an ordered pair *of objects*. Thus, the second coordinate of an ordered pair must be an object; it cannot be a plurality of two or more objects.

Fortunately, there is a natural way to assign sense to the mentioned expressions.[7] Suppose we want to talk about the ordered pair of P and aa.

[6] For more examples from natural language, see Section 9.4.

[7] For details and a more general treatment, see Appendix 11.A.

Consider all ordered pairs of the form $\langle P, a \rangle$, where $a \prec aa$. The plurality *pp* of such ordered pairs can be used to represent the desired but problematic pair $\langle P, aa \rangle$. To see that this representation works, observe first that the representing plurality *pp* is well defined, and second, that the representation uniquely determines *P* and *aa* and thus does all the work that the problematic ordered pair was meant to do. To wit: given the mentioned plurality *pp* of ordered pairs, *P* can be retrieved as the unique object that figures as the first coordinate of all the ordered pairs *pp*, and *aa* can be retrieved as the plurality of objects each of which figures as the second coordinate of one of these ordered pairs. In light of this, it is unproblematic to use the familiar notation '$\langle P, aa \rangle$' as a suggestive shorthand for the mentioned plurality *pp*.

With this convention in place, we can proceed to state the interpretation of a plural predicate *P* as a bunch of ordered pairs, $\langle P, aa \rangle$, $\langle P, bb \rangle$, et cetera—keeping in mind that this bunch will be a superplurality, as it is a bunch of pluralities. We can then say, informally, that an atomic plural predication, such as *Ptt*, is true if and only if *tt* stands for a plurality that appears as second coordinate in one of the pairs $\langle P, aa \rangle$, $\langle P, bb \rangle$, et cetera.

To provide a formal clause capturing these truth conditions, we need to define a notion of interpretation capable of representing the target interpretation of plural predicates as well as other non-logical expressions. As it turns out, this can be done by letting an interpretation be a superplurality *iii*.[8] Its components will be a domain *dd* and an interpretation function *fff*, which now consists of a superplurality. A variable assignment remains a plurality *ss*. Let us use 'are among' to indicate the membership relation between a plurality *xx* and a superplurality *xxx*. So, loosely speaking, *xx* are among *xxx* just in case *xx* are one of the pluralities comprising *xxx*. We are finally in a position to state the satisfaction clause for an atomic plural predication:

$$iii \vDash Ptt\,[ss] \text{ if and only if } [\![tt]\!]_{iii,ss} \text{ are among } [\![P]\!]_{iii,ss}$$

where $[\![tt]\!]_{iii,ss}$ is a plurality and $[\![P]\!]_{iii,ss}$ a superplurality.

Let us now turn to the intensional approach to predication, which takes properties rather than pluralities (or superpluralities) as semantic values of predicates. To interpret plural predicates, we need *plural properties*, that is, properties that (if instantiated) are instantiated by many things *jointly*. By

[8] Again, see Appendix 11.A for technical details.

contrast, *singular properties* (if instantiated) are instantiated by many things *separately*. For example, the property of cooperating is plural while that of being human is singular.

Suppose that plural properties are objects.[9] Then, given any plural property π, we could obtain an interpretation function ff^{+} for $\mathcal{L}_{\text{PFO+}}$ by simply adding the pair $\langle P, \pi \rangle$ to the interpretation function ff considered above. For an atomic predication, we would therefore have:

$$ii \models Ptt\,[ss] \text{ if and only if } [\![tt]\!]_{ii,ss} \text{ have } [\![P]\!]_{ii,ss}$$

which is perfectly analogous to the satisfaction clause for singular predication on the intensional approach.

However, there is significant pressure to reject the supposition that plural properties are objects. For this supposition is subject to a variant of a Russell-style argument put forth by Williamson (2003), and it clashes with a plural version of Cantor's theorem.[10] Let us briefly comment on the last claim. As discussed in Section 3.5, the instance of Plural Cantor concerned with the universal plurality entails that there are more pluralities than objects. If properties are objects, it follows that there cannot be a plural property corresponding to every plurality. But this may seem implausible. If there are some things, why shouldn't there also be a plural property had by them and only them?

The natural reaction is to postulate a type distinction between objects and properties.[11] Thus, we may think of properties as higher-level entities, which, following Frege, we call concepts. This view leads us to combine the resources of plural logic with those of second-order logic. The resulting system is an extension of PFO+ where quantification into predicate position is allowed for both singular and plural predicates. For example, these generalizations are legitimate:

(7.8) $\exists F \neg Ftt$

(7.9) $\exists F \forall vv\, Fvv$

(7.10) $\forall vv \exists F\big(Fvv \wedge \forall uu(Fuu \rightarrow \forall v(v \prec uu \leftrightarrow v \prec vv)\big)$

[9] In the context of plural logic and its semantics, this assumption is endorsed by Hossack 2000 and McKay 2006.

[10] See Florio 2014c for a detailed exposition of these arguments.

[11] This approach is endorsed, for example, by Oliver and Smiley and by Yi (see footnote 3 on p. 131).

In fact, they are also provable with the help of the appropriate instances of this scheme of comprehension for plural concepts:

(PSO-Comp) $$\exists F \forall xx(Fxx \leftrightarrow \varphi(xx))$$

where F does not occur free in φ. We assume this scheme as well as its polyadic analogues.

Postulating a type distinction between objects and properties avoids the two problems mentioned above. The Russell-style argument is blocked for essentially the same reason Russell's paradox was blocked in the simple set theory discussed in Chapter 4. That is, owing to the sortal distinctions between individuals, pluralities, and properties (now concepts), the key condition driving the paradoxical argument cannot even be formulated (see Williamson 2003, Section IX). Moreover, if properties are no longer objects, it is consistent to hold *both* that there are more pluralities than objects *and* that there is a plural property corresponding to every plurality. So there is no clash with the plural version Cantor's theorem.

How can we accommodate the view that predicates stand for concepts in the plurality-based model theory? As in the case of superplurals, we cannot supply the interpretation of predicates by simply adding some ordered pairs to the interpretation function. Concepts are not objects. So we cannot represent the fact that the semantic value of S is the singular concept X, and the semantic value of the predicate P is the plural concept Y, by means of $\langle S, X \rangle$ and $\langle P, Y \rangle$. For these are not proper pairs.

In fact, there is a way to represent not only single such "pairs" but also many of them simultaneously. We resort to concepts of an even higher level than X and Y. To represent a bunch of pairs of the form $\langle S, X \rangle$, we use a second-level concept $\mathbf{R}$ with two argument places, one open to objects and the other open to first-level concepts, such that $\mathbf{R}(S, X)$ just in case $\langle S, X \rangle$ is one of the target bunch of pairs.[12] By quantifying over the appropriate sort of higher-level concepts, we can then define a notion of interpretation capturing the informal idea that an atomic plural predication Ptt is true if and only if Fxx, where xx is the plurality for which tt stands and F is the plural concept interpreting P.

[12] This approach, in its current version, assumes that concepts are individuated extensionally. That assumption can be dispensed with, if desired. One option is to adopt suitable modalized comprehension principles of the form $\exists F \Box \forall x(Fx \leftrightarrow \varphi(x))$.

7.6 The problem of choice

We have surveyed a number of ways in which the model theory for plural logic may be developed. On the one hand, we could opt for a set-based model theory. We argued in Section 7.4 that this model theory is no more problematic than the usual set-based model theory for first-order logic, at least from a purely logical point of view and assuming the relevant reflection principle. On the other hand, we could opt for an extensional or intensional version of a plurality-based model theory. The formulation of this style of model theory has led us to introduce richer expressive resources. The model theory for PFO was carried out in PFO+, as it construed interpretations as pluralities and thus relied on a plural predicate to characterize the notion of truth in an interpretation. The model theory for PFO+ was carried out in a metalanguage including either superplural quantification or quantification over concepts. As noted, the ascent to more expressive metalanguages is not an accident but a robust Tarskian phenomenon, which we explore systematically in Chapter 11.

The existence of multiple model theories presents us with a problem: how are we to choose between the available options? This *problem of choice*, as we will call it, is connected with large and difficult philosophical questions. So we will not attempt to reach a final verdict. We do, however, aim to paint as complete a picture as possible of the considerations that are relevant to solving the problem.

It is useful to classify the available options on the basis of whether or not they use certain resources. Does a given option use plural resources? And does it use conceptual resources? The table below summarizes the alternatives and indicates some of the authors who have adopted them.

	no plural resources	plural resources
no conceptual resources	Link and other linguists, Quine, Resnik	Boolos, Hossack, McKay, Rayo
conceptual resources	Higginbotham & Schein, Florio	Oliver & Smiley, Yi, Rayo & Yablo, Williamson, Linnebo

Three of the four alternatives are exemplified by model theories discussed above: the set-based model theory uses neither plural not conceptual resources; the extensional version of the plurality-based model theory uses plural but not conceptual resources; the intensional version of plurality-based model theory uses both plural and conceptual resources.

However, we have not presented any option that uses only conceptual resources. Let us briefly mention two such options.[13]

The first imitates the eliminative strategy of Higginbotham and Schein discussed in Chapter 6. They analyze predication in terms of events and eliminate pluralities in favor of concepts. The same resources (events and concepts) could be put to use in developing a model theory for PFO+. The second way to construct a model theory that uses only conceptual resources imitates another eliminative strategy discussed in Chapter 6. We have seen that there is a translation of PFO+ into MSOL+. Similar results can be obtained for extensions of these theories. In particular, the systems needed to formulate the extensional or intensional version of plurality-based model theories for PFO+ can be interpreted in a fragment of higher-order logic containing a few layers of concepts. These conceptual resources can be used to develop a model theory for PFO+. What's more, it can be shown that the resulting model theory delivers the same relation of consequence as the plurality-based model theory (see Florio 2014a).

Recall that the set-based model theory can deliver the same relation of consequence as the plurality-based model theory for PFO, assuming the appropriate reflection principle (Section 7.4). This continues to hold when we add plural predicates to the object language. It follows that all the options considered are on a par, as far as logic is concerned. If we are to solve the problem of choice, we must look beyond the logic.

In light of this, someone with a purely instrumental view of the semantics may deny that there is a problem of choice after all. If model theory is a mathematical tool whose sole purpose is to characterize a relation of consequence for a given language, then any option is just as good as any other option—at least in the case at hand. And if an option must be selected, one might well select the simplest and most economical one. However, if we want model theory to be not just a mathematical study of logical relations but a study of possible meanings of natural and formal languages, then there might be further constraints capable of discriminating between the options.

[13] The existence of all these alternatives may be unsurprising: owing to the interpretability results presented in Part II, we know that it is possible in principle to imitate any of the relevant systems in any other of those systems.

7.7 Absolute generality as a constraint

One such constraint is absolute generality. Since this issue will receive a detailed discussion in Chapter 11, we limit ourselves to some brief remarks. Absolutely general quantification seems possible; for example, it seems possible to assert that the empty set has no elements, none whatsoever. What would be the domain of an absolutely general quantifier? It cannot be a set, since standard set theory recognizes no universal set. Perhaps it can be a plurality (though see Chapter 12). For example, plural comprehension implies that there are some objects *uu* such that:

$$\forall x(x \prec uu \leftrightarrow x = x)$$

This plurality includes absolutely everything there is and hence every set. Using *uu* as a domain, the plurality-based model theory is able to represent an absolutely general interpretation of the quantifiers. Thus, absolute generality—if there is such a thing—can serve as a constraint that narrows down the options, ruling out the set-based model theory.

Could this conclusion be resisted by using some non-standard set theory that does accept a universal set?[14] We might for example use New Foundations (see Forster 2019) or some "logical" notion of set (or property) of the kind developed in Fine 2005c and Linnebo 2006. Any such maneuver would only shift the bump in the carpet. Recall the plural version of Cantor's theorem. In the context of traditional plural logic, this implies that there are more pluralities than objects. Since any plurality can be used to interpret a one-place predicate—namely by letting the predicate be true of precisely these objects—the theorem means that there are more interpretations of the predicate than can be represented by objects—no matter what kind of singular representation one chooses, including any non-standard conception of set. If we want our model theory to capture every possible interpretation of a predicate, then the plural version of Cantor's theorem serves to rule out any form of semantic singularism cashed out in first-order terms.

While absolute generality promises to be a powerful weapon against any such form of semantic singularism, it is perfectly consistent with a model theory formulated within higher-order logic. If we accept higher-order logic,

[14] Class theories such as von Neumann-Bernays-Gödel and Morse-Kelley take a major step in this direction by accepting a class of all sets. But there can be no class of all objects, since a class is prohibited from being an element of a set or class.

there is good reason to accept a universal concept. This corresponds to the following instance of second-order comprehension:

$$\exists F \forall x (Fx \leftrightarrow x = x)$$

The universal concepts can then serve to represent a domain of quantification containing absolutely all objects. As indicated in Section 7.6, it can be shown more generally that every plurality-based model has an isomorphic model described with purely conceptual resources (Florio 2014a).

So absolute generality does not single out the plurality-based model theory as the only viable option. Put in terms of our diagram on p. 144, absolute generality takes out the upper left-hand quadrant, but is neutral with respect to the three remaining options.

7.8 Parity constraints

There are additional constraints one might impose on the model theory. One might require that certain theoretical and empirical desiderata be satisfied. For example, one might want the metatheory to be ontologically parsimonious or the representation of possible meanings to be psychologically plausible. Moreover, one might want to be able to integrate the model theory for PFO+ with the model theory for a language including a broader class of expressions (such as generalized quantifiers, adverbs, and modalities). As we have seen in Chapter 5, a similar thought is an important motivation behind the analysis of plurals in terms of individual mereology favored by some linguists.

In this section, we want to examine some constraints that demand some form of parity between the language being analyzed and the language used to analyze it. We therefore call them *parity contraints*. We consider four such constraints.

First, we remarked above that the truth conditions of the plurality-based model theory have a pleasing homophonic feel. This contrasts with the set-based model theory whose truth conditions appear more artificial: they equate truth in an interpretation to facts about sets. Could we rule out semantic singularism, and any model theory based on purely conceptual resources, by assuming *homophonicity* as a constraint? We need to be careful. Since we are dealing with model theory, not truth theory, it is not really an option to require homophonicity across the board. This would conflict with the aim of letting non-logical expressions be reinterpreted from model to model.

Note that this also applies to more restricted requirements of homophonicity, such as the following *deflationist constraint on predication*:

(7.11) $\forall y$('F' is true of $y \leftrightarrow Fy$)

It states that a predicate can be said to be true of whatever it can be truly predicated of. Friederike Moltmann cites this constraint in support of plural reference and notes that it reflects "what is generally considered an important condition on a semantic theory, namely that the object language be included in the metalanguage" (2016, 108). A plural predicate can be truly predicated of pluralities and hence these must be the semantic values of the terms with which the predicate combines. Is this a reasonable constraint? Again, we need to be careful. Since we are dealing with model theory, our ability to enforce the constraint is limited. While the constraint may be plausible for truth theory, it is not an option for model theory.

Our question, then, is whether one might capture the spirit of homophonicity and of the deflationist constraint on predication through a requirement that is applicable to model theory. A second constraint does just that—by requiring that the semantic value of an expression of the object language be given by an expression of the same type in the metalanguage. Let us call this the principle of *type preservation*. This constraint is less demanding than the previous two, requiring only that the type of an expression be preserved by its semantic value. On the plausible assumption that the relevant types are determined by the logico-linguistic categories of PFO+, type preservation solves the problem of choice in favor of the intensional version of the plurality-based semantics. Type preservation rules out the extensional version of plurality-based model theory because this model theory interprets predicates as superpluralities rather than concepts. Moreover, it rules out the remaining model theories because they fail to preserve the type of plural terms, interpreting them as objects or concepts rather than pluralities.

However, the power of type preservation comes with far-reaching, revisionary consequences. It can often be illuminating to analyze expressions of one type using resources from other types. An example is the analysis of tense in terms of explicit reference to, and quantification over, moments of time. Another example is the usual set-based model theory for ordinary first-order languages, which interprets predicates by means of sets rather than concepts and therefore violates the principle of type preservation.

Indeed, much of the set-based model theory employed in linguistics and mathematics would have to be rewritten. Especially in the case of linguistics, it is unclear whether this can be done successfully while holding on to the principle of type preservation. We have no guarantee that the array of types needed to regiment natural languages can be systematically incorporated in a unified and adequate model theory. A case in point is that of modals: these expressions are usually interpreted using possible worlds, not primitive modalities in the metalanguage.

A final parity constraint concerns the *modal profile* of plural terms. These are generally thought to be rigid.[15] That is, the following principles are supposed to hold. Let E be an existence predicate (paraphrasable as $\exists zz\, zz \approx \ldots$), and let $\Box$ stand for metaphysical necessity. Then:

(RGD$^+$) $$\Box \forall x\, \forall yy\, (x \prec yy \rightarrow \Box(Eyy \rightarrow x \prec yy))$$

(RGD$^-$) $$\Box \forall x\, \forall yy\, (x \nprec yy \rightarrow \Box(x \nprec yy))$$

Informally, the principles state that if this object is one of those objects, then necessarily, whenever those objects exist, this object is one of them. Similarly, if this object is not one of those objects, then necessarily this object is not one of them. But unlike plural terms, predication is not rigid, as illustrated by the next example.

(7.12) John is tall but might not have been.

Now consider this constraint concerning modal profile: semantic values should have the same modal profile as the expressions of which they are semantic values. It would follow that predicates cannot have superpluralities as semantic values, as the latter but not the former are rigid. Similarly, plural terms could not have concepts as semantic values, as the former but not the latter are rigid. Thus, the constraint helps with the problem of choice by eliminating the off-diagonal options in our diagram (p. 144).

Once again, the constraint can be challenged. For example, Kripke semantics for modal logic is widely regarded as illuminating, despite using semantic values with the "wrong" modal profile. Although sets possess their members necessarily, in Kripke semantics they are successfully used as semantic values of predicates. The key is to allow these semantic values to vary from world to world.

[15] See Chapter 10 for details and a defense.

7.9 Conclusion

We have discussed various constraints that may help us choose among the available model theories for plural logic. First, we have absolute generality, which rules out the options in the upper left-hand quadrant of our diagram. Then, we have two parity constraints applicable to model theory: type preservation—which selects an option in the bottom right-hand quadrant—and modal profile—which rules out the off-diagonal options. Putting everything together, only the bottom right-hand quadrant remains.

However, neither of these parity constraints was found to be absolutely compelling. So there seems to be no simple solution to the problem of choice. What is required to make progress, it seems to us, is greater clarity on what model theory is supposed to do. On a minimal conception of the role of model theory, such as the one espoused by an instrumentalist about semantics, the existence of several, equally good options is perfectly acceptable. This is less so if model theory is supposed to capture certain features of the "true nature" of our expressive resources. In that case, one might insist on a model theory that is not only extensionally but also intensionally correct. This is especially important when we lack independent means of determining the correct extension. If so, we can establish an extensionally correct theory by relying on an intensionally correct one. These considerations lend further support to the option in the bottom right-hand quadrant. This option will play a role in Chapter 11, where we again discuss semantic matters in the presence of absolute generality.