2022

Mathematics

[Generic]

(B.Sc. Second Semester End Examination-2022)

PAPER-MTM GE201

(Differential equation & Differential Calculus - II)

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Group-A

[Differential equation]

1. Answer any four questions: **4x2= 8**

a) Find the Wronskian of the set $\{1-x, 1+x, 1-3x\}$

b) Write the principle of superposition of linear differential equation.

c) Solve the equations $\frac{dx}{myz}=\frac{dy}{nzx}=\frac{dz}{pxy}$

d) Find the general solution of the ODE $\frac{d^2x}{dx^2}-\frac{2dy}{dx}+10y=0$

e) Find the particular solution of the differential equation $\left(D^2+3aD-4a^2\right)y=0 \quad y(0)=1, y^1(0)=2$

f) Define ordinary point and singular point of a differential equations $a_0(x)\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)y=0$

g) Show that x=1 and x=3 are the ordianry point and the singular points of the equation

$$x(3-x)\frac{d^2y}{dx^2}-(3-x)\frac{dy}{dx}+5xy=0$$

2. Answer any two questions: **2x5 = 10**

a) Solve $(D^2+3D+2)y=e^{2x}Sinx$

b) Solve $(D^2+4D+1)y=x^2-2x+2$ by using the method of undetermined coefficients.

c) Solve by the method of variation of parameters

$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}-\frac{1}{x^2}y=x+Sixx,\ (x>0)$ It being given that $y=x$ and $y=1/x$ are two linearly independent solutions of the associated homogeneous differential equation.

3. Answer any one question: **1x10 = 10**

a) i) Find the series solution of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ around the point $x=1$

ii) Solve $\frac{x\,dx}{z^2-2yz-y^2}=\frac{dy}{y+z}=\frac{dz}{y-z}$

iii)Solve $(D^4-n^4)y=0$

b) i) Find the general solution of the $ODE\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^{3x}$

ii) Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+4y=3\sin x$

Group B

[Differential Calculus – II]

4. Answer any four questions: **4x2= 8**

a) State Rolle's theorem.

b) Find from definition, the partial derivative of the function $f(x,y)=x^2\log y$ w.r. to x at the point (1,2)

c) Show that the function $f(x)=\frac{1}{x^2-2x+1)^{3/2}}$ has no derivative at $\gamma=1$

d) Find at x=1 what values of x the function $f(x)=12x^5-45x^4+40x^3+1\forall x\in\mathbb{R}$ has maximum or minimum.

e) If $f^1(x)=g'(x)$ in *[a,b],* then show that $f(x)=g(x)$ is equal or not.

f) State Schwarz theorem on commutatine property of mixed derivative.

g) What is directional derivative?

h) Verify $\underset{(x,y)\to(0,0)}{Lt}\frac{2x^2y}{x^2+y^2}$ exist or not.

i) If $f(x) = \tan x$, then $f(0) = 0 = f(\pi)$. Is Rolle's theorem applicable to $f(x)$ in $[0, \pi]$

5. Answer any two questions: 2x5 = 10

a) Let
$$f(x,y) = xy \text{ if } |x| \geq |y|.$$
$$= -xy \text{ if } |x| < |y|.$$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

b) Staqte and prove Lang range Mean value theorem.

c) Expand sin x as an infinite series in power of x by the use of Maclaurin's theorem.

6. Answer any one question: 1x10 = 10

a) i) State and prove Euler's theorem on homogeneous function of two variables. 3+2

Prove that $\underset{x \to 0}{Lt} \underset{y \to 0}{Lt} \frac{x-y}{x+y} \neq \underset{y \to 0}{Lt} \underset{x \to 0}{Lt} \frac{x-y}{x+y}$.

ii) Verify Rolle's theorem for $f(x) = 1 - |x-1|$ on [0,2]. 5

b) i) For the function
$$f(x,y) = \frac{x^3 + y^3}{x - y}, x \neq y$$
$$= 0, x = y$$

Prove that $f(x,y)$ is not continuous at (0,0) but $\frac{df}{dx}, \frac{df}{dy}$ exist at (0,0) 5

ii) Find the maximum and minimum value of $\sin x\,(1 + \cos x)$ in $[0, 2\pi]$ 5